Stony Brook University
The Graduate School

Doctoral Defense Announcement

Abstract

Potts Model and Generalizations: Exact Results and Statistical Physics

By

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The q-state Potts model is a spin model that has been of longstanding interest as a many body system in statistical mechanics. Via a cluster expansion, the Potts model partition function Z(G,q,v), defined on a graph G=(V,E), where V is the set of vertices (sites) and E is the set of edges (bonds), is expressed as a polynomial in terms of q and a temperature-dependent Boltzmann variable v. An important special case (v=-1) is the zero-temperature Potts antiferromagnet, for which Z(G,q,-1)=P(G,q), where P(G,q) is the chromatic polynomial, counting the number of ways of assigning q colors to the vertices of graph G such that no two adjacent vertices have the same color.

A natural generalization is to consider this model in a generalized external field that favors or disfavors spin values in a subset $I_s=\{1,\ldots,s\}$ of the total set of q-state spin values. In this dissertation, we calculate the exact partition functions of the generalized Potts model Z(G,q,s,v,w), where w is a field-dependent Boltzmann variable, for certain families of graphs. We also investigate its special case, viz. Z(G,q,s,-1,w)=Ph(G,q,s,w), which describes a weighted-set graph coloring problem.

Nonzero ground-state entropy (per lattice site), $S_0 \neq 0$, is an important subject in statistical physics, as an exception to the third law of thermodynamics and a phenomenon involving large disorder even at zero temperature. The q-state Potts antiferromagnet is a model exhibiting ground-state entropy for sufficiently large q on a given lattice graph. Another part of the dissertation is devoted to the study of ground-state entropy, for which lower bounds on slabs of the simple cubic lattice and exact results on homeomorphic expansions of kagomé lattice strips are presented. Next, we focus on the structure of chromatic polynomials for a particular class of graphs, viz. planar triangulations, and discuss implications for chromatic zeros and some asymptotic limiting quantities.

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