

# Generalized isometries in superspace

A Dissertation Presented

by

**Itai Ryb**

to

The Graduate School

in Partial Fulfillment of the Requirements

for the Degree of

**Doctor of Philosophy**

in

**Physics**

Stony Brook University

August 2010

UMI Number: 3426525

All rights reserved

INFORMATION TO ALL USERS

The quality of this reproduction is dependent upon the quality of the copy submitted.

In the unlikely event that the author did not send a complete manuscript and there are missing pages, these will be noted. Also, if material had to be removed, a note will indicate the deletion.



UMI 3426525

Copyright 2010 by ProQuest LLC.

All rights reserved. This edition of the work is protected against unauthorized copying under Title 17, United States Code.



ProQuest LLC  
789 East Eisenhower Parkway  
P.O. Box 1346  
Ann Arbor, MI 48106-1346

**Stony Brook University**

The Graduate School

**Itai Ryb**

We, the dissertation committee for the above candidate for the Doctor of Philosophy degree, hereby recommend acceptance of this dissertation.

Martin Roček - Advisor

Professor, Department of Physics and Astronomy

Peter van Nieuwenhuizen - Chairperson of defense

Professor, Department of Physics and Astronomy

Paul Grannis

Professor, Department of Physics and Astronomy

Leonardo Rastelli

Professor, Department of Physics and Astronomy

Leon Takhtajan

Professor, Department of Mathematics, Stony Brook University

This dissertation is accepted by the Graduate School

Lawrence Martin

Dean of the Graduate School

Abstract of the Dissertation

# Generalized isometries in superspace

by

**Itai Ryb**

**Doctor of Philosophy**

in

**Physics**

Stony Brook University

2010

$N = (2, 2)$  supersymmetric models are of interest for mathematicians and physicists and have been used extensively as a tool for the investigation of generalized Kähler geometry. In the sigma-model approach, it is convenient to formulate and manipulate sigma-models in superspace where essential geometric properties are captured by the generalized Kähler potential which gives rise to bihermitian geometry description. Recent developments in differential geometry show that one can also characterize these targets using structures that interpolate between complex and symplectic geometry and are defined on the sum  $T \oplus T^*$ .

The research work that will be presented here extends the set of known superspace tools for the manipulation of bihermitian / generalized Kähler geometries, namely, the gauging of isometries along directions that mix chiral and twisted chiral or semichiral multiplets.

Other results that will be presented relate to possible  $N = (4, 4)$

supersymmetry in semichiral models and sigma models formulation  
on the sum  $T \oplus T^*$ .

To my parents

# Contents

List of Figures	viii
List of Tables	ix
Acknowledgements	x
<b>1 Introduction</b>	<b>1</b>
1.1 Motivation: The string action . . . . .	1
1.2 Supersymmetric sigma-models . . . . .	3
1.3 $N = (1, 1)$ supersymmetry in superspace . . . . .	3
1.4 Complex Geometry . . . . .	5
1.5 $N = (2, 2)$ supersymmetry . . . . .	8
1.5.1 $N = (2, 2)$ supersymmetry in superspace . . . . .	10
1.5.2 Constrained Superfields . . . . .	11
1.5.3 Matter Actions . . . . .	13
1.5.4 Gates-Hull-Roček, Take One $[J_+, J_-] = 0$ . . . . .	14
1.5.5 Gates-Hull-Roček, Take two: $[J_+, J_-] \neq 0$ . . . . .	14
1.6 Gauging Isometries . . . . .	16
1.6.1 Bosonic model . . . . .	16
1.6.2 Gauging in $N = (2, 2)$ superspace . . . . .	17
1.6.3 $N = (1, 1)$ superspace and components . . . . .	20
1.6.4 T-duality and Buscher rules . . . . .	22
1.7 $d$ -isometries and the $O(d, d, \mathbb{Z})$ group . . . . .	25
1.8 Generalized Complex Geometry . . . . .	27
1.8.1 Operations on $T \oplus T^*$ . . . . .	27

<b>2</b>	<b>Some new results towards <math>O(d, d, \mathbb{Z})</math> covariant formalism</b>	<b>32</b>
2.1	$O(d, d, \mathbb{Z})$ transformations of Complex Structures . . . . .	33
2.1.1	The transformation a complex structure under factor- ized duality . . . . .	34
2.1.2	The transformation of a complex structure under an el- ement of $O(d, d, \mathbb{Z})$ . . . . .	36
2.2	Nonlinear sigma-models on $T \oplus T^*$ . . . . .	38
2.2.1	The generalized kinetic term . . . . .	39
2.2.2	The generalized WZ-term . . . . .	39
2.2.3	Field-redefinitions and frames for the action . . . . .	40
2.2.4	The 0-frame . . . . .	40
2.2.5	The $g$ -frame . . . . .	42
2.2.6	$\Sigma$ -frames . . . . .	43
2.2.7	$O(d, d, \mathbb{Z})$ transformations . . . . .	43
2.2.8	(Partial) Invariance of the WZ-term in the Courant frame	45
2.2.9	The transformed generalized sigma-model . . . . .	46
2.2.10	Linear and fractional-linear transformations . . . . .	47
2.2.11	Lift to $N = (1, 1)$ superspace . . . . .	48
<b>3</b>	<b>Gauging and dualities along generalized isometries</b>	<b>49</b>
<b>4</b>	<b>Nonabelian multiplets</b>	<b>81</b>
<b>5</b>	<b>Abelian LVM action</b>	<b>95</b>
<b>6</b>	<b><math>N = (4, 4)</math> Supersymmetry for semichirals</b>	<b>106</b>
	<b>Bibliography</b>	<b>132</b>



# List of Figures

1.1 The worldsheet embedded in  $D$  dimensional target . . . . . 1

# List of Tables

1.1	Constrained $N = (2, 2)$ superfields . . . . .	11
2.1	Frames for the off shell sigma-model on $T \oplus T^*$ . . . . .	44

# Acknowledgements

This is a wonderful opportunity to thank:

Peter van Nieuwenhuizen, for teaching and inspiration,

Ulf Lindström, for support, hospitality, and patient guidance,

Warren Siegel, and Barry McCoy for many encouraging conversations,

Vladimir Korepin, for introducing me to some beautiful ideas,

Martin Roček, for all of the above, as well as his Supervision and a roof.

and all students and postdocs at Uppsala and Stony Brook for their friendship and support - material, and moral.

# Chapter 1

## Introduction

### 1.1 Motivation: The string action

String theory [1–3] is a compelling framework for the quantization of gravity on the same footing as all other forces of nature, that is, as a microscopic theory of spin 2 particles. It overcomes the hurdles of nonrenormalizability in four dimensions by promoting point particles propagating along a timeline to strings which are maps from a two-dimensional *worldsheet*,  $\Sigma$ , to  $D$ -dimensional target space  $M^D$

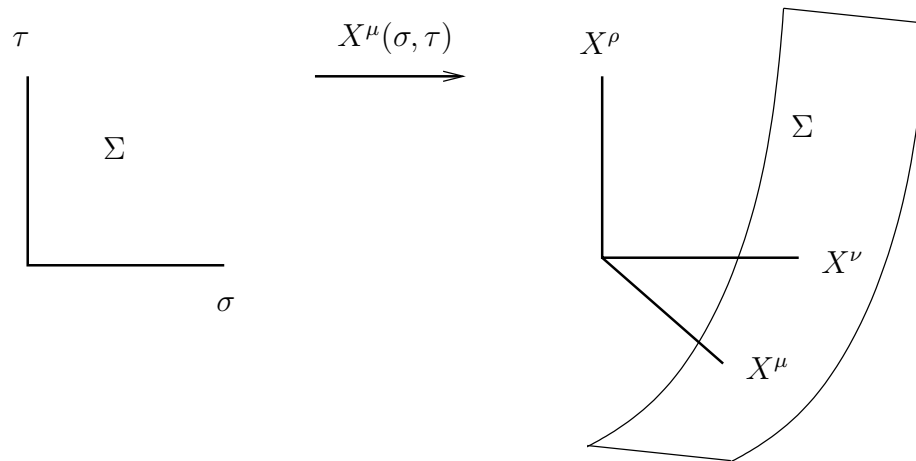


Figure 1.1: The worldsheet embedded in  $D$  dimensional target

The string action is a generalization of that of the particle, namely, the

Nambu-Goto action which minimizes this surface in  $M^D$  reads

$$S = -T \int_{\Sigma} d^2\sigma \sqrt{(X' \cdot \dot{X})^2 - \dot{X}^2 (X')^2}, \quad (1.1)$$

where we introduce the string tension  $T = \frac{1}{2\pi\alpha'}$ . This action is, classically, equivalent to the Polyakov action where a worldsheet metric  $h_{\alpha\beta}$  is introduced

$$S = -\frac{T}{2} \int_{\Sigma} \sqrt{-h} h^{\alpha\beta} \eta_{\mu\nu} \partial_{\alpha} X^{\mu} \partial_{\beta} X^{\nu}, \quad (1.2)$$

In the conformal gauge  $h = \text{diag}(-1, 1)$  this gives a sigma-model

$$S = -\frac{T}{2} \int d^2\sigma \eta_{\mu\nu} \partial X^{\mu} \bar{\partial} X^{\nu}, \quad (1.3)$$

where  $\partial = \partial_{\sigma} + \partial_{\tau}$ .

The two dimensional string is therefore embedded in the  $D$ -dimensional target space through *nonlinear sigma-model*; which is a field theory whose fields are the coordinates of a Riemannian manifold.

Strings need not propagate on flat targets only. An interesting class of backgrounds that will be considered here are due to NS-NS sector and admit a metric  $G_{\mu\nu}(X)$ , an antisymmetric tensor which is a torsion potential  $B_{\mu\nu}(X)$  and a dilaton coupling the Ricci scalar which gives an expansion parameter. These modes are matched with the massless spectrum of the closed string. A string propagating in such a background is therefore subject to the action

$$S = -\frac{T}{2} \int_{\Sigma} d^2\sigma \sqrt{-h} (h^{\alpha\beta} G_{\mu\nu}(X) + \epsilon^{\alpha\beta} B_{\mu\nu}(X)) \partial_{\alpha} X^{\mu} \partial_{\beta} X^{\nu} + \alpha' \Phi R^{(2)}. \quad (1.4)$$

Nonlinear sigma-models also appear in other branches of physics such as statistical physics where they appear as continuum limits with the for a spin system which preserves the target metric.





















































































































































































































































































































