

Generalized isometries in superspace

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by

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Abstract of the Dissertation

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$N = (2, 2)$ supersymmetric models are of interest for mathematicians and physicists and have been used extensively as a tool for the investigation of generalized Kähler geometry. In the sigma-model approach, it is convenient to formulate and manipulate sigma-models in superspace where essential geometric properties are captured by the generalized Kähler potential which gives rise to bihermitian geometry description. Recent developments in differential geometry show that one can also characterize these targets using structures that interpolate between complex and symplectic geometry and are defined on the sum $T \oplus T^*$.

The research work that will be presented here extends the set of known superspace tools for the manipulation of bihermitian / generalized Kähler geometries, namely, the gauging of isometries along directions that mix chiral and twisted chiral or semichiral multiplets.

Other results that will be presented relate to possible $N = (4, 4)$

supersymmetry in semichiral models and sigma models formulation
on the sum $T \oplus T^*$.

To my parents

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Chapter 1

Introduction

1.1 Motivation: The string action

String theory [1–3] is a compelling framework for the quantization of gravity on the same footing as all other forces of nature, that is, as a microscopic theory of spin 2 particles. It overcomes the hurdles of nonrenormalizability in four dimensions by promoting point particles propagating along a timeline to strings which are maps from a two-dimensional *worldsheet*, Σ , to D -dimensional target space M^D

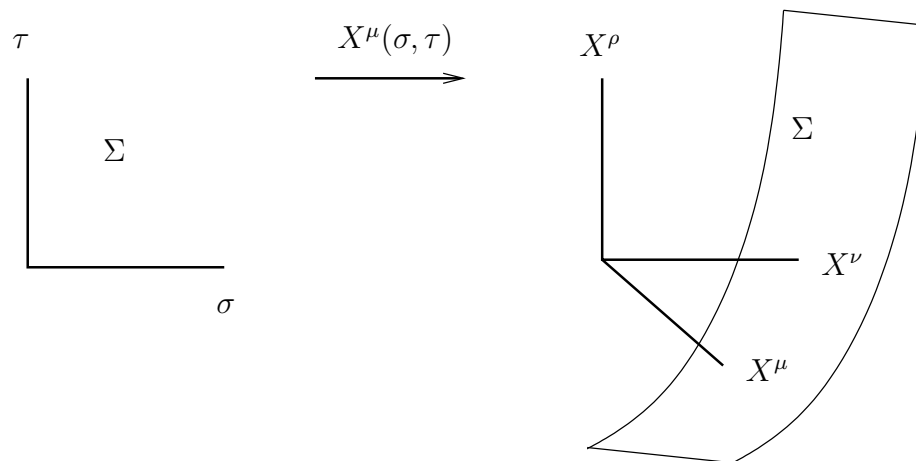


Figure 1.1: The worldsheet embedded in D dimensional target

The string action is a generalization of that of the particle, namely, the

Nambu-Goto action which minimizes this surface in M^D reads

$$S = -T \int_{\Sigma} d^2\sigma \sqrt{(X' \cdot \dot{X})^2 - \dot{X}^2 (X')^2}, \quad (1.1)$$

where we introduce the string tension $T = \frac{1}{2\pi\alpha'}$. This action is, classically, equivalent to the Polyakov action where a worldsheet metric $h_{\alpha\beta}$ is introduced

$$S = -\frac{T}{2} \int_{\Sigma} \sqrt{-h} h^{\alpha\beta} \eta_{\mu\nu} \partial_{\alpha} X^{\mu} \partial_{\beta} X^{\nu}, \quad (1.2)$$

In the conformal gauge $h = \text{diag}(-1, 1)$ this gives a sigma-model

$$S = -\frac{T}{2} \int d^2\sigma \eta_{\mu\nu} \partial X^{\mu} \bar{\partial} X^{\nu}, \quad (1.3)$$

where $\partial = \partial_{\sigma} + \partial_{\tau}$.

The two dimensional string is therefore embedded in the D -dimensional target space through *nonlinear sigma-model*; which is a field theory whose fields are the coordinates of a Riemannian manifold.

Strings need not propagate on flat targets only. An interesting class of backgrounds that will be considered here are due to NS-NS sector and admit a metric $G_{\mu\nu}(X)$, an antisymmetric tensor which is a torsion potential $B_{\mu\nu}(X)$ and a dilaton coupling the Ricci scalar which gives an expansion parameter. These modes are matched with the massless spectrum of the closed string. A string propagating in such a background is therefore subject to the action

$$S = -\frac{T}{2} \int_{\Sigma} d^2\sigma \sqrt{-h} (h^{\alpha\beta} G_{\mu\nu}(X) + \epsilon^{\alpha\beta} B_{\mu\nu}(X)) \partial_{\alpha} X^{\mu} \partial_{\beta} X^{\nu} + \alpha' \Phi R^{(2)}. \quad (1.4)$$

Nonlinear sigma-models also appear in other branches of physics such as statistical physics where they appear as continuum limits with the for a spin system which preserves the target metric.

1.2 Supersymmetric sigma-models

To obtain a realistic spectrum which allows also fermionic modes one must also include worldsheet fermions ψ^μ ¹. Adding fermions to the Polyakov action we find

$$S = -\frac{T}{2} \int d^2\sigma \partial^\alpha X^\mu \partial_\alpha X_\mu + \bar{\psi}^\mu \rho^\alpha \partial_\alpha \psi_\mu . \quad (1.5)$$

This model exhibits a symmetry with fermionic parameter mixing bosons and fermions, known as *supersymmetry* (e. g. [17]) that acts as follows

$$\delta X^\mu = \epsilon \psi^\mu , \quad \delta \psi^\mu = \rho^\alpha \partial_\alpha X^\mu \epsilon , \quad (1.6)$$

and closes on-shell to give translations:

$$[\delta_{\epsilon^1}, \delta_{\epsilon^2}] = 2\bar{\epsilon}_1 \rho^\alpha \epsilon_2 \partial_\alpha . \quad (1.7)$$

Since there are two generators for this symmetry of opposite chiralities this symmetry is $N = (1, 1)$ supersymmetry.

1.3 $N = (1, 1)$ supersymmetry in superspace

A convenient way to write down manifestly $N = (1, 1)$ supersymmetric actions is to endow space with anticommuting directions θ^α , $\alpha = \pm$ such that supersymmetry transformations are translations. In two dimensions, one can introduce real spinors

$$C_{\alpha\beta} = -C_{\beta\alpha} = -C^{\alpha\beta}, \quad C_{+-} = i, \quad \theta_\alpha = \theta^\beta C_{\beta\alpha}, \quad \theta^\alpha = C^{\alpha\beta} \theta_\beta . \quad (1.8)$$

The supersymmetry transformations $\delta_\epsilon = [-i\epsilon^\alpha Q_\alpha, \cdot]$ shifts the worldsheet coordinates

$$Q_\alpha = i \frac{\partial}{\partial \theta^\alpha} + \theta^\beta \partial_{\beta\alpha}, \quad \delta_\epsilon \sigma^\pm = -i\epsilon^\pm \theta^\pm, \quad \delta_\epsilon \theta^\pm = \epsilon^\pm \quad (1.9)$$

¹This also mends the inconsistency due to tachyonic mode in the bosonic model

