General instructions: Three problems are given. You should do any two. Each problem counts 20 points and the solution should typically take less than 45 minutes. Use one exam book for each problem and label it carefully with your name, the name of the problem's author and the date. You may use a one page help sheet, a calculator, and with the proctor's approval a foreign language dictionary. No other materials may be used.

Classical Mechanics I (McCarthy)

A uniform ladder of length $l$ leans against a smooth (no friction) wall. The floor is also smooth (no friction) and the ladder makes an angle of $\theta_0$ with the floor when the ladder is installed at rest.

a) (7 points) Before the ladder leaves the wall, express the equations of motion of the ladder in terms of a single generalized coordinate.

b) (6 points) Find a constant of the motion.

c) (7 points) Find the angle $\theta$ at which the ladder leaves the wall.

Classical Mechanics II (Likharev)

A system with 1 degree of freedom has the Lagrangian function $L = \frac{m}{2} \dot{x}^2 - \frac{k}{2} x^2 + \frac{\mu}{2} x^2 \dot{x}^2$.

a) (8 points) Find an integral of motion, i.e. a function of $x$ and $\dot{x}$ which is conserved during the motion.

b) (12 points) Find the amplitude ($A$) dependence of the oscillation frequency of the system for the limiting case of $\mu A^2 \to 0$.

Classical Mechanics III (Likharev)

Two thin, elastic rods of identical length $l$ ($d << l$) have different cross-sections as shown in the Fig. below ($t << d$). Both rods are bent within the plane $x-z$ by the same bending force. Find the ratio of the bending deformations $s_a/s_b$ of the two rods.