STONY BROOK UNIVERSITY

DEPARTMENT OF PHYSICS AND ASTRONOMY

Comprehensive Examination, September 2, 2009

General Instructions: Twelve problems are given; you should do any four. If you do more than four problems, you must choose which four should be graded, and only submit those four.

Each problem counts 20 points, and the solution should typically take less than 45 minutes.

Some of the problems spill onto two pages. Make sure you do all the parts of each problem you choose.

Use one exam book for each problem, and label it carefully with the problem topic and number and your name.

You may use a one page help sheet, a calculator, and with the proctor's approval, a foreign language dictionary. No other materials may be used.

Some potentially useful information:

- The atomic mass of hydrogen is 1.00794 amu.
- The atomic mass of helium is 4.002602 amu.
- 1 amu is 1.66×10^{-27} kg.
- $c = 2.998 \times 10^8$ ms^{-1}.
- $G = 6.673 \times 10^{-11}$m^{3}kg^{-1}s^{-2}.
- The solar luminosity is 3.85×10^{26} W.
- The mass of the Sun is 1.989×10^{30} kg
- The radius of the Sun is 7.00×10^8 meters
- $\hbar = 1.055 \times 10^{-34}$ J
- $e = 1.602 \times 10^{-19}$ C
- $k_B = 1.38 \times 10^{-23}$ J/K

Astronomy 1

Pulsars, pulsating radio sources, were first observed in 1967 by Jocelyn Bell Burnell. At first there was considerable speculation on the origin of such pulsating sources, but the observed characteristics soon allowed astronomers to deduce the basic components. This problem address the process of deduction. The following are observed properties of pulsars (from Carroll and Ostlie, 2nd ed.):

- Most pulsars are found to have periods between 0.25 and 2 s, with an average period of 0.795.
- Pulsars have extremely well-defined pulse periods, allowing them to challenge the accuracy of the best atomic clocks.
- The periods of all pulsars increase very gradually over time.

a. (5 points) Suppose a postulated model consisted of two Sun-like stars orbiting each other. If the orbital period were taken as the average pulsar period, what would be the average separation? What does this result allow one to conclude about the viability of this model?

b. (5 points) Suppose another postulated model consisted of a pair of neutron stars orbiting each other. If both neutron stars have a mass of 1.4 $M_\odot$ and again assuming the average pulsar period, what would be the average separation? If the neutron stars are very close, what other physical effects are important and what will be the consequence for the evolution of the orbits. Is this consistent with observed pulsar properties?

c. (5 points) Another conceivable pulsar model is a pulsating star. Estimate the pulsation period of a star in the simplifying assumption of constant density. Evaluate your result for a neutron star of mass 1.4 $M_\odot$ and radius of 12 km. Is this a viable model? Why or why not?

d. (5 points) Yet another pulsar model is a rapidly rotating star. For a spherical star of radius R (i.e., neglecting deformation due to rotation) obtain an expression for the minimum rotation period. Evaluate your result for a neutron star of mass 1.4 $M_\odot$ and radius of 12 km. Is this a viable model? Why or why not?

Astronomy 2

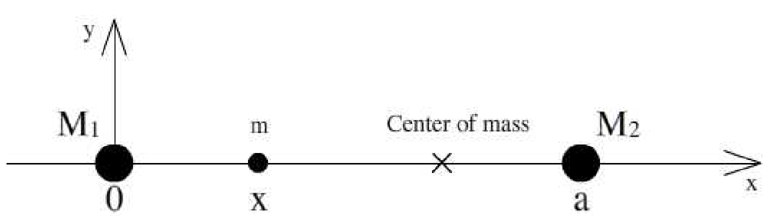
This problem concerns properties of neutron stars.

a. (10 points) Estimate the Roche limit for a star in the vicinity of a black hole. You may use Newtonian gravity. Assume that since the neutron star is deformed that the effective tidal force is twice as strong as assuming the neutron star remains spherical. Apply it to a neutron star of mass $M_N = 1.4\ \mathrm{M}_\odot$ and radius $R_N = 15$ km $\simeq 7GM_N/c^2$. Determine for what black hole masses that tidal disruption of the neutron star occurs outside the last stable circular orbit of the black hole.

b. (10 points) Determine the maximum spin rate of a uniformly rotating neutron star of mass $M_N = 1.4\ \mathrm{M}_\odot$ and non-spinning radius $R_N = 15$ km. You may use the following assumptions: Use Newtonian gravity, assume the gravitational potential of the star to be that of a non-spinning point mass, and assume the polar radius of the star does not change with rotation. Show that the maximum spin rate is proportional to the square root of the average density of the non-spinning star, and show that the ratio of the equatorial radius to the polar radius at the maximum spin frequency is 3/2.

Astronomy 3

Consider two objects with mass M_1 and M_2 in a circular orbit with an angular velocity ω. M_1 and M_2 are on the x-axis and M_1 is at the origin in a rotating coordinate system with M_1 and M_2 at the same angular velocity ω (see Figure). The separation between M_1 and M_2 is a.



a. (5 points) Consider a test particle with a negligible mass m at the coordinate $(x, 0)$. Besides the gravitational force from M_1 and M_2, it should feel the centrifugal force $F_c = m\omega^2 r$, where r is the distance from the center of mass. Write the effective potential energy $\Phi_{\rm eff}$ at $(x, 0)$ by assuming that the "centrifugal energy" can be defined as

$$U = -\int F_{\rm c} dr \ .$$

Use G as gravitational constant.

Sketch the shape of $\Phi_{\rm eff}$ along x-axis (i.e., plot $\Phi_{\rm eff}$ as a function of x, with $y = 0$), and answer how many local maxima exist along x-axis. The positions of these local maxima are called Lagrange points.

b. (5 points) Assume that M_1, M_2, m are the Sun, Earth, and Moon, respectively. The gravitational pull of the Sun on the Moon is stronger than that of the Earth. Explain why the Moon can remain in a stable orbit about the Earth.

c. (5 points) Lagrange points can be derived by solving the equations:

$$\begin{aligned} \frac{\partial \Phi_{\rm eff}}{\partial x} &= 0 \\ \frac{\partial \Phi_{\rm eff}}{\partial y} &= 0 \ . \end{aligned}$$

For simplicity, we consider only motion along the x direction (the first equation above) at $y = 0$. When $M_1 \ll M_2$, the center of mass should be very close to M_2. Hence, two Lagrange points around M_1 should satisfy $|x|/a \ll 1$. Use this approximation and Kepler's third law, $\omega^2 = G\left(M_1 + M_2\right)/a^3$, and derive the distance of the two Lagrange points from M_2 (this is called the Jacobi radius).

d. (5 points) Lagrange points set the maximum radius of a dwarf galaxy orbiting around a large galaxy. If the size of a dwarf galaxy exceeds the Jacobi radius, it cannot remain bound by self-gravity. Consider the Large Magellanic Cloud (LMC) and Milky Way (MW) as M_1 and M_2, respectively, and calculate the maximum radius of LMC. The masses of LMC and MW are $10^{10} M_\odot$ and $5 \times 10^{11} M_\odot$, respectively. The LMC is about 50 kpc away from MW, and has the diameter of about 9 kpc. Can LMC remain bound?

Astronomy 4

Consider a toy model star with a radial density profile $\rho(r) = \rho_c(1 - r/R)$, where $\rho_c \equiv \rho(r = 0)$ is the central density and R is the stellar radius. Assume that at $r = R$ the surface pressure and temperature vanish: $P(R) = T(R) = 0$.

a. (3 points) Determine an expression for the central density in terms of R and the total stellar mass M.

b. (4 points) Use hydrostatic equilibrium and the boundary conditions to find an expression for the pressure $P(r)$ in the form $P_c \times$ function(r/R). Evaluate the central pressure P_c numerically for the Sun's M (1.99×10^{33} g) and R (6.96×10^{10} cm).

c. (4 points) Prove that the mean atomic weight per particle (including charged ions, protons, and electrons) for fully ionized gas that is X parts hydrogen (H) and Y parts helium (He) by mass, where $X + Y = 1$, is given by $\mu = (2X + 0.75Y)^{-1}$.

d. (3 points) Compute the central temperature T_c in this model, assuming that the pressure is that of an ideal gas with composition $X = 0.7$ and $Y = 0.3$ by mass (as may be appropriate for a zero-age main sequence star).

e. (3 points) Repeat (d) for a pure helium $Y = 1$ star. (These exist, and are called zero-age horizontal branch stars!) Explain qualitatively the difference you find (i.e., why is the star hotter or colder?).

f. (3 points) Real stars are more centrally concentrated than those in this toy model. Do you expect their central temperatures (given M and R) to be higher or lower than those in the toy model? How about the central pressures?

AMO 1

Ultracold atoms trapped in standing waves of laser light (optical lattices) can serve as model systems for the behavior of electrons in solids. This problem asks some fundamental questions about atoms in a one-dimensional lattice.

a. (5 points) Describe the fundamental mechanism that generates the trapping potential $V(x)$ for the atoms in the optical lattice. By treating the atoms as classical harmonic oscillators (resonance at ω_0, neglect damping), determine how the optical potential $V(x)$ varies with the frequency ω and amplitude $E_0 \cos kx$ ($k = 2\pi/\lambda$) of the optical field.

b. (5 points) The dynamics of the atoms in the lattice depends on its depth V_0, which can be tuned widely. If it is very deep, the atoms are localized in individual, separated wells; if it is very shallow (or completely absent) its effect on the motion of the atoms is very small. Sketch the lowest three bands (i.e., energy versus quasimomentum) for three cases: (1) vanishing lattice depth, (2) intermediate lattice depth and (3) large lattice depth (each well can be approximated by a harmonic confinement).

c. (5 points) For vanishing lattice depth, calculate the kinetic energy of atoms at the edge of the first Brillouin zone (the so-called recoil energy E_r). For a finite lattice depth V_0, calculate the harmonic-oscillator frequencies $n\omega_{ho}$ in each well. What is the width of the harmonic-oscillator ground state for a ^{87}Rb atom ($m = 1.4 \times 10^{-25}$kg) in a $20E_r$ deep lattice at $\lambda = 1064$ nm?

d. (5 points) When the lattice is "tilted" e.g., by adding gravity along x, periodic oscillations (Bloch oscillations) of the atoms in the lattice can be observed. Explain the mechanism that leads to these oscillations (consider the dynamics of an atom in the lowest band).

AMO 2

A 1 mW beam of monochromatic light is incident perpendicular to a lossless plane mirror which reflects 99% of the light, and the transmitted beam then strikes an ideal power meter.

a. (3 points) Draw a sketch of this setup. What is the reading on the meter?

b. (7 points) A second identical mirror is placed a distance L away from, and parallel to the first one, between it and the power meter. Draw a second sketch of this new setup. Now the meter reading has a periodic (but not sinusoidal) dependence on L. Make a plot of this dependence, explain why there should be maxima and minima, and where they should be. Be sure to indicate their heights, depths, and widths. Also, be sure to label both axes **quantitatively**.

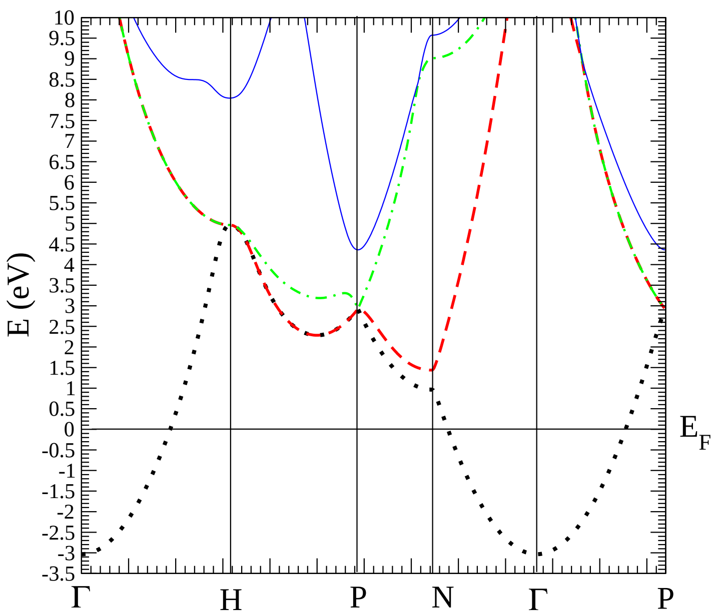
c. (6 points) For certain values of L the meter reading is higher than in part (a) above. Explain in some detail how this can happen. Your answer should explain what is wrong with the following argument: "If the first mirror reflects 99% of the light, how could the second mirror possibly recover that lost light and cause it to hit the power meter."

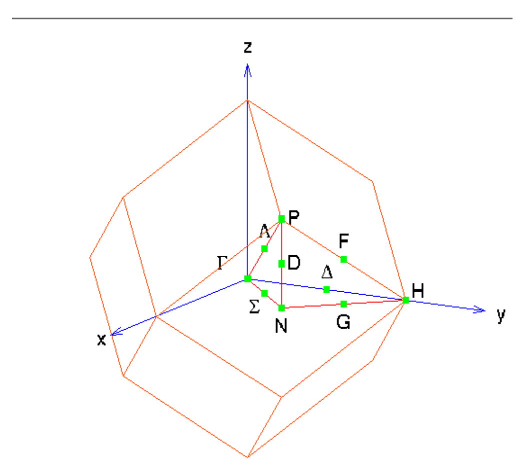
d. (4 points) Because the meter indeed reads higher with the second mirror at certain places than the reading without it as in part (a), one might wonder if energy is being conserved. This is because it seems that the light initially reflected by the first mirror cannot reach the power meter. Explain what really happens to the reflected beam.

Condensed Matter 1

In the left side figure below, the band structure of a body-centered cubic (BCC) monoatomic crystal at T=0 is shown. It has been calculated using density functional theory (DFT), in the local density approximation (LDA). Therefore this is not the experimental band structure, although it is not too far from the experimental one, because for this material the LDA approximation works very well. Different line styles represent different bands. The right side figure shows the first Brillouin zone of a BCC crystal. Indicated are the points that define the paths used to compute the band structure. The Cartesian coordinates of some of these points are:

- $N = \frac{2\pi}{a}(1/2, 1/2, 0)$
- $P = \frac{2\pi}{a}(1/2, 1/2, 1/2)$
- $H = \frac{2\pi}{a}(0, 0, 1)$





The Fermi Energy (E_F) is indicated by the horizontal line at 0.0 eV.

a. (2 pt) What kind of electronic material is it (i.e., insulator, metal, semimetal or semiconductor)? (Explain, briefly).

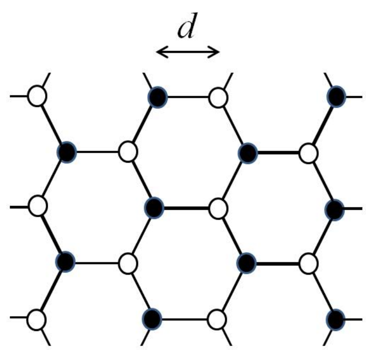
b. (6 pt) Estimate the lattice constant of the underlying cubic Bravais lattice, using the free electron approximation.

c. (6 pt) How many valence electrons does each atom have? How did you get to this number? (I will not accept a guess as valid answer.)

d. (6 pt) What is the energy of the $V_{10\bar{1}}$ Fourier component of the potential seen by the electrons? (Explain how you obtain this number.)

Condensed Matter 2

The figure shows a rectangular fragment of a two dimensional crystal. The layered form of boron nitride has this structure, if we imagine peeling off a single layer. The filled circles represent boron, the open circles represent nitrogen, and the length of each bond is d.



a. (5 points) Choose a primitive unit cell. What are the primitive translation vectors $\vec{a}$ and $\vec{b}$ that correspond to your choice? How many atoms and what types are in this primitive cell?

b. (5 points) What are the corresponding reciprocal lattice vectors $\vec{a^*}$ and $\vec{b^*}$? Draw the shape of the (two-dimensional) first Brillouin zone. You may use the standard symmetrical (Wigner-Seitz) construction, or any other correct representation.

c. (5 points) To simplify things, when the atoms vibrate, let us forbid any motion in the direction perpendicular to the $x-y$ plane of the picture. Then there are no z-polarized vibrations. On a graph of ω *vs.* Q, along any particular direction in reciprocal ($\vec{Q}$) space, indicate the expected form of the phonon dispersion $\omega_i(Q)$ for all branches i.

d. (5 points) The only thermal excitations of this insulating two dimensional crystal are the phonons. At low temperature, only the low frequency phonons are relevant, and their density of states is $D(\omega) \propto \omega^p$, for some power p. What is the power p of the density of states for this two-dimensional hexagonal crystal? Explain your reasoning - no credit for a guess, even if it is correct.

Nuclear 1

In the 1956 paper of Lee and Yang on parity violation, you can read that the angular distribution of β radiation of Co^{60} is of the form

$$I(\theta)d\theta = (\text{constant})(1 + \alpha\cos\theta)\sin\theta d\theta. \tag{1}$$

Here, θ is the angle between the orientation of the parent nucleus and the momentum of the outgoing electron.

a. (4 points) Why does $\alpha \neq 0$ imply parity violation?

b. (3 points) Give the weak decay process in terms of nucleons and in terms of quarks (and leptons).

c. (10 points) In the appendix of the paper of Lee and Yang it is shown that α is of order v/c with v the speed of the electron in the rest frame of the nucleus. Using that the relevant effective vertex for this weak decay is given by

$$\frac{g^2\cos\theta_c}{8M_W^2}\bar{\nu}\gamma_\mu(1-\gamma_5)e\bar{d}\gamma_\mu(1-\gamma_5)u \tag{2}$$

show that the parity violating part of this vertex is of order v/c. You may use the following explicit representation of the γ-matrices

$$\gamma_k = \begin{pmatrix} 0 & -i\sigma_k \\ i\sigma_k & 0 \end{pmatrix}, \qquad \gamma_0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \qquad \gamma_5 = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}. \tag{3}$$

d. (3 points) If you cannot answer c) you can still earn points by explaining the symbols g, θ_c, and M_W in the effective vertex. Also explain why M_W appears as $1/M_W^2$.

Nuclear 2

You are to design a π^0 spectrometer to detect and measure the decay $\pi^0 \to \gamma\gamma$. For simplicity, we will assume that the pions initially are moving with 1 GeV/c momentum in the lab frame along the z axis. The mass of a neutral pion is $m_\pi = 0.139\frac{GeV}{c^2}$.

a. (6 points) Calculate the minimum and maximum possible *lab* energy of the photons observed in the lab. Under what conditions are these produced?

b. (6 points) Assume instead that the decay axis of the π^0 is perpendicular to the initial direction of its flight. Under this condition what is the lab energy and angular separation of the two photons?

c. (8 points) Using the information from parts (a) and (b), specify a detector technology, detector size, and resolution (position and energy resolution) appropriate for the task. NOTE: You do not need to catch every possible decay of these pions in your device, but you must catch some.

High Energy 1

a. (6 points) In the Standard Model, the Higgs boson is introduced to *spontaneously* break the electroweak symmetry. Explain in words what *spontaneous* symmetry breaking means in this context.

   Explain why we want the breaking to be spontaneous instead of explicitly adding the W and Z gauge boson masses to the Lagrangian.

b. (7 points) Besides giving mass to the electroweak gauge bosons, the Higgs field also gives a Dirac mass to the charged fermions.

   (i) Explain how (in words).

   (ii) Explain the difference between a Majorana and a Dirac fermion and the difference between a Majorana and a Dirac mass term.

   (iii) Could the electron be a Majorana fermion? Explain why.

c. (7 points) At LHC the best channel for discovery of a light Higgs boson is $pp \to H \to \gamma\gamma$

   (i) Draw the dominant Feynman diagrams contributing to this process. (Hint: the dominant Feynman diagrams in the production and decay of Higgs bosons are all one-loop diagrams).

   (i) Why is this channel better than the tree level process $pp \to q\bar{q} \to H \to bb$?

High Energy 2

This question concerns decay modes and branching fractions of various known and hypothetical particles. (For any kinematically allowed decay, you can assume the *decay products* have $m = 0$ unless stated otherwise.)

a. (2 points) What are the allowed decay modes of a W boson?

b. (4 points) The W coupling to leptons is independent of lepton species. What are the decay branching fractions for each of the modes in part (a)?

c. (2 points) What are the allowed decay modes of a Z boson?

d. (4 points) The Z coupling to a fermion/antifermion pair is proportional to $g_V - g_A\gamma_5$ with γ_5 one of the Dirac matrices, $g_V = I_3 - 2q\sin^2\theta_W$ and $g_A = I_3$. Here I_3 is the weak isospin component, q is the charge and $\sin^2\theta_W \approx 0.23$ is the square of the sign of the weak mixing angle. What are the decay branching fractions for each of the modes in part (c)?

e. (4 points) What are the (Standard Model) allowed decay modes of a top quark?

f. (4 points) A leptoquark (LQ) is a hypothetical, fundamental particle which carries both lepton and quark flavors. LQs undergo two body decays which conserve charge, lepton number, lepton flavor and quark flavor (and of course the usual conservation of energy and angular momentum). Current experimental bounds say $M_{LQ} > 200$ GeV. What are the possible quantum numbers (spin, charge and flavors) for LQs, and what are their allowed decay modes? (Assume LQ decay only to particles, not to antiparticles.)