

PHY 541 Advanced Statistical Mechanics

Syllabus

1. Basic Principles

1. Thermodynamics
 - 1.1 Macroscopic; Extensive and Intensive
 - 1.2 Equilibrium
 - 1.3 The four laws of thermodynamics
2. Statistical mechanics
 - 2.1 Statistical philosophy
 - 2.2 The microcanonical ensemble
 - 2.3 The canonical ensemble
 - 2.4 The grand canonical ensemble
3. Quantum statistical mechanics
 - 3.1 The relation of classical to quantum statistical mechanics
4. Quantum field theory

2. Stability, existence and uniqueness

1. Classical stability
 - 1.1 Catastrophic potentials
 - 1.2 Conditions for stability
 - 1.3 Superstability
 - 1.4 Multi-species interactions
2. Quantum stability
 - 2.1 Results
 - 2.2 Proofs
3. Existence and uniqueness of thermodynamic limit
 - 3.1 Boundary conditions
 - 3.2 Existence and uniqueness in the canonical ensemble
 - 3.3 Existence and uniqueness in the grand ensemble
4. Continuity and differentiability of the pressure
5. Discussion

3. Mayers' Virial expansions and Groeneveldt's theorems

1. Second virial coefficient
2. Mayer's first theorem
3. Mayer's second theorem

4. Nonnegative potentials and Groeneveldt's theorems

4.1 The alternation of sign of b_l

4.2 Upper and lower bounds on b_l

4.3 Bounds on the radius of convergence

5. Convergence of virial expansions

6. Counting of Mayer graphs

4. Ree-Hoover virial expansion and hard spheres,

1. The Ree-Hoover expansion

2. The Tonks gas

3. Hard sphere virial coefficients $B_2 - B_8$

4. Virial coefficients for $k > 8$

5. Convergence of the virial expansion and approximate equations of state

5. Hard spheres and discs at high density and freezing

1. Numerical methods

1.1 Hard spheres

1.2 Hard discs

2. Packing problem for hard spheres

3. Free volume theory for high densities

4. Freezing as a competition of free energies

5. Mixtures of hard spheres

6. Power law and positive potentials

1. Scaling considerations

2. Powerlaw potentials

3. Positive potentials

7. Potentials with attraction

1. Attractive square well

2. Kac potentials

3. Lattice gas

4. Ising model

4. Zeroes of partition functions

6. Series expansions

7. Critical exponents and Pade approximates

8. The Ising model in two dimensions; summary of results

1. $H = 0$
2. $H \neq 0$
3. Impure Ising models

9. Ising model in two dimensions; exact solution at $H = 0$

1. Dimers
2. Ising partition function
3. Correlations and spontaneous magnetization

10. Scaling theory and critical phenomena

1. Scaling limit and scaling functions
2. Scaling laws for critical exponents
3. Universality

11. Integrable models; Boltzmann weights

1. Transfer matrices
2. Vertex models
 - 2.1 Star triangle equation
 - 2.2 6 vertex model
 - 2.38 vertex model
3. Spin models
 - 3.1 Star triangle equation
 - 3.2 Chiral Potts model Boltzmann weights
 - 3.3 Chiral Potts from 6-vertex
4. Face models
 - 4.1 Star triangle equation
 - 4.2 RSOS models

12 Functional equations for eigenvalues and free energies

1. 8 vertex
 - 1.1. Root of unity
 - 1.2 Generic
 - 1.3 Free energy
 - 1.4 Excitation spectrum
2. 6 Vertex
 - 2.1 Root of unity
 - 2.2 Generic

- 2.3 Free energy
- 2.4 Excitation spectrum
- 3. Chiral Potts
 - 3.1 Functional equation
 - 3.2 Free energy
 - 3.3 Excitation spectrum

13. Order parameters and correlation functions

- 1. 6 vertex model
- 2. 8 vertex model
- 3. Chiral Potts
- 4. RSOS