Group Theory

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PHY 680, Fall 2016; MWF: 12:00 – 12:53 (P117)

We have chosen to discuss those subjects in group theory which we have found to be central for modern theoretical physics. We give proofs of most theorems, but the emphasis is on applications.

1. We begin with finite group theory. First we define some basic concepts. Then we present theorems on matrix representations. As an application we derive the Dirac matrices in n dimensions, in Euclidean and Minkowski space, and the charge conjugation matrices, as well as their reality properties. As another application, we construct the multiplets of normal modes of polyhedra and some molecules such as buckyballs.

2. Then we define the simple Lie algebras SU(n), SO(n), Sp(n) and F_4, G_2, E_6, E_7, E_8 using particular explicit matrix representations. We construct the corresponding Lie groups, and discuss covering groups and the various real forms.

3. From these defining matrix representations of simple groups we read off the roots, Cartan generators, and we introduce weights, and all other representations. This leads to Dynkin diagrams, conjugacy classes of Lie algebras, and the spinor representations for SO(n).

4. Next we introduce Young tableaux for SU(n), SO(n) and Sp(n). We first state the rules how to obtain them and how to multiply them, and give many examples. Then we use the group S_n to explain the construction of Young tableaux.

5. We also consider non-semisimple groups, in particular the Poincaré group and derive their unitary irreducible representations and Casimir operators by using the theory of induced representations.

6. We discuss some topological properties of compact and noncompact Lie groups.

7. Then we construct coset manifolds, coset vielbeins and connections, the coset measure (Haar measure), torsions and curvatures, covariant Lie derivatives and spherical harmonics on coset manifolds.

8. Finally we consider some interesting generalizations: Kac-Moody algebras, superalgebras, and Virasoro algebras.

Prerequisites: For the mathematical part, none; the course is just fun and is accessible to any graduate student. For some of the applications, some familiarity with classical gauge field theories (Yang-Mills fields coupled to Dirac fermions) is useful.

Evaluation: There will be homework. The final exam consists of a written and an oral exam, and the grade will be based on these exams and to a lesser degree on the homework.

Office hours: Appointment can be made after each class.

Literature: We use H. Georgi, *Lie Algebras in Particle Physics*, and W. Ledermann, *Introduction to the Theory of Finite Groups*. Additional notes will be handed out in class.