Homework Set 4. Due Monday October 1, 2012

1. This problem is to check if the Clausius Massotti relation agrees with experimental data.

   a. First, using simple physics, derive the relation

   \[ \epsilon = 1 + nV(\epsilon_{sp} - 1) \]  

   for the dielectric constant a medium that consists out of spheres with volume \( V \), a dielectric constant \( \epsilon_{sp} \) and density \( n \).

   b. Check whether relation (1) or the Clausius-Massotti relation best describes the experimental results for air and pentane

   \[
   \begin{array}{|c|c|}
   \hline
   \text{Pressure (atm)} & \epsilon \\
   \hline
   20 & 1.0108 \\
   40 & 1.0218 \\
   60 & 1.0333 \\
   80 & 1.0439 \\
   100 & 1.0548 \\
   \hline
   \end{array}
   \]

   Dielectric constant for air at 292 K.

   \[
   \begin{array}{|c|c|c|}
   \hline
   \text{Pressure (atm)} & \text{Density (g/cm}^3) & \epsilon \\
   \hline
   1 & 0.613 & 1.82 \\
   10^3 & 0.701 & 1.96 \\
   4 \times 10^3 & 0.796 & 2.12 \\
   8 \times 10^3 & 0.865 & 2.24 \\
   12 \times 10^3 & 0.907 & 2.33 \\
   \hline
   \end{array}
   \]

   Dielectric constant for Pentane at 303 K.

2. Starting from

\[
d\vec{B} = \frac{I}{c} \frac{d\vec{l} \times (\vec{x} - \vec{x}')}{|\vec{x} - \vec{x}'|^3}
\]

for the magnetic field at \( \vec{x} \) from current \( I d\vec{l} \) at \( \vec{x}' \), show that for a closed loop carrying current \( I \) the magnetic field at point \( P \) is

\[
\vec{B} = \frac{I}{c} \nabla \Omega
\]

with \( \Omega \) the solid angle of the loop at \( P \).

3. A Spherical shell of radius \( R \) with a uniform surface charge density \( \sigma \) is rotated about an axis with constant angular velocity \( \vec{\omega} \).

   a. Find the magnetic field inside and outside the sphere.

   b. How should you wind a wire around the nonrotating sphere such that the magnetic field inside the sphere is uniform?

4. In class we derived the equation of motion for the magnetic moment

\[
\frac{d\vec{M}}{dt} = \vec{\omega} \times \vec{M} - \frac{d}{dt} \frac{e^2}{6m_0c^2} \sum_\epsilon \vec{r}_\epsilon \times \vec{B}
\]

Assume \( \vec{B} \) (and \( \Omega_L \)) is parallel to the \( z \)-axis. Solve for \( M_x \), \( M_y \) and \( M_z \) for given initial conditions of \( \vec{M} \).