trace of a tensor \[ A^\mu = A_0^\mu + A_1^\mu + A_2^\mu + A_3^\mu \]

contraction of indices

Four-dimensional Kronecker delta

\[ \delta^\mu_\nu = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \]

\[ g_{\mu\nu} = g_{\mu\nu} \delta^\nu_\nu \]

\[ g^{\mu\nu} = g^{\mu\nu} \]

Lorentz transformations more abstract

\[ A^\mu = A_0^\mu + A_1^\mu + A_2^\mu + A_3^\mu \]

\[ A_0^\mu = L_0^\nu A_\nu \]

\[ A_1^\mu = L_1^\nu A_\nu \]

\[ A_2^\mu = L_2^\nu A_\nu \]

\[ A_3^\mu = L_3^\nu A_\nu \]

\[ L_0^\nu A_\nu = A_0^\mu g_{\mu\nu} A_1^\nu + A_1^\mu g_{\mu\nu} A_2^\nu + A_2^\mu g_{\mu\nu} A_3^\nu \]

\[ = L_0^\nu A_\nu g_{\mu\nu} L_0^\mu g_{\mu\nu} A_1^\mu \]

\[ = A_0^\mu g_{\mu\nu} L_0^\nu g_{\mu\nu} g_{\mu\nu} L_0^\mu g_{\mu\nu} A_1^\mu \]

true for all \( A \)

\[ q_{\mu\nu} L_\nu^\mu \frac{\partial}{\partial x_\mu} q^{\mu\nu} L_\mu^\nu = q_{\mu\nu} q^{\mu\nu} \]

\[ L_\nu^\mu q_{\mu\nu} L_\mu^\nu = q_{\mu\nu} q^{\mu\nu} \]