28d) Invariant tensors

Transformation of $g$

\[ g_{\mu' \nu}' = L_{\mu}^{\ \mu'} L_{\nu}^{\ \nu'} g_{\mu \nu} = (L g L^T)_{\mu' \nu}' \]

\[ L^T g L = g \implies L = g L^{-1} g \]

\[ L g L^T = L g g L^{-1} g = g \implies g \text{ is invariant} \]

\[ \delta_{\mu' \nu}' g_{\mu' \nu}' g_{\mu' \nu}' \text{ is invariant} \]

If a tensor is symmetric in one frame, it is symmetric in all frames.

\[ A'_{\mu' \nu}' = L_{\mu}^{\ \mu'} L_{\nu}^{\ \nu'} A_{\mu' \nu}' \]
\[ = L_{\nu}^{\ \nu'} L_{\mu}^{\ \mu'} A_{\mu' \nu}' \]
\[ = L_{\mu}^{\ \mu'} L_{\nu}^{\ \nu'} A_{\mu' \nu}' = A_{\mu' \nu}' \]

frame change

\[ A_{\mu \nu} = - A_{\nu \mu} \implies A'_{\mu' \nu}' = - A'_{\nu' \mu}' \]

$A_{\mu \nu}$ is tensor of rank $2$

$A_{\mu \nu}$ $\nu$ $\mu$ $\mu$ $\nu$ $\nu$ $\mu$

antisymmetry or symmetry in each pair of indices is preserved