Completely anti-symmetric tensor

Any $$\epsilon_{\mu\nu\rho\sigma}$$ is anti-symmetric under the exchange of each pair of indices. This is preserved by Lorentz transformations.

$$\Rightarrow$$ \( \epsilon_{\mu\nu\rho\sigma} = 0 \) if two indices are the same.

$$\Rightarrow$$ only one independent component

\( \epsilon_{0123} \)

**Notation**

\( \epsilon_{0123} = 1 \)

\( \epsilon_{\mu\nu\rho\sigma} = 1 \) if \( \mu\nu\rho\sigma \) is an even permutation of \( 0123 \)

\( = -1 \) if \( \mu\nu\rho\sigma \) is an odd permutation of \( 0123 \)

\( \epsilon_{\mu_1\nu_1\rho_1\sigma_1} = \epsilon_{\mu_1\nu_2\rho_1\sigma_2} \epsilon_{\mu_2\nu_1\rho_1\sigma_2} \epsilon_{\mu_2\nu_2\rho_2\sigma_2} \epsilon_{\mu_3\nu_3\rho_3\sigma_3} \epsilon_{\mu_4\nu_4\rho_4\sigma_4} \)

\( = 1 \) \( \epsilon_{0123} = \det L \)

\( \Rightarrow \) \( \epsilon \) is a pseudo-tensor

**Dual tensor** \( A^* \mu_1\nu_1\rho_1\sigma_1 = \frac{1}{6} \epsilon_{\mu\nu\rho\sigma} A_{\mu\nu} \)

Called Hodge star

\( A^* \) is a pseudoscalar e.g. \( A_{\mu\nu} = x_\mu x_\nu \)

Then it always contains a time component.