a) Use of Green's Function

Suppose that we know \( G(\vec{r}, \vec{r}') \) for a conducting surface \( S_c \) and \( S_b \) with given charge \( \phi_{c,1} = 0 \) and \( Q_{bc} = 0 \).

Conducting surface

Then we know the potential for arbitrary \( \phi_{c,1} \) and charge density \( \rho \).

**Proof**

Consider \( \Sigma \int_{S_c} \left[ G(\vec{r}, \vec{r}') \nabla \cdot \chi(\vec{r}') - \nabla G(\vec{r}, \vec{r}') \cdot \nabla \chi(\vec{r}') \right] d\vec{s}' \)\d\vec{s}' // \text{ inward normal}

on \( S_c \) \( G(\vec{r}, \vec{r}') = 0 \)

on \( S_b \) \( \int_{S_b} \nabla G(\vec{r}, \vec{r}') \cdot \chi(\vec{r}') \ d\vec{s}' \)

\[ = -\phi_c \int_{V} \nabla \cdot G(\vec{r}, \vec{r}') \ dV \]

\[ = -\frac{1}{4\pi} \phi_c Q_c \]

and vanishes at surface

on \( S_c \) \( \int_{S_c} \nabla G(\vec{r}, \vec{r}') \cdot \chi(\vec{r}') \ d\vec{s}' \)

\[ = -\frac{1}{4\pi} \phi_c Q_c \]

\( \text{ minus sign cancels because of inward normal} \)

on \( S_b \) \( \int_{S_b} G(\vec{r}, \vec{r}') \nabla \cdot \chi(\vec{r}') \ d\vec{s}', \)

\[ = \phi_c(\vec{r}) \left( -\int_{V} \nabla \cdot \chi(\vec{r}') \ dV \right) \]