Boundary condition \( \phi(x, y = 0) = V \)

(19)

\[ \sum_{n \neq 0} c_n \frac{\sin \frac{n \pi x}{a}}{a} = V \]

\[ \frac{\sin \frac{n \pi x}{a}}{a} \quad \text{odd} x \quad \text{even} x \]

\[ \Rightarrow c_n = 0 \quad \text{for} \ n = \text{odd} \]

\[ c_n = \int_{-a}^{a} V \cos \frac{n \pi x}{a} \, dx \quad \text{for} \ n = \text{even} \]

\[ = \frac{2}{a^2} V \int_{0}^{a} \sin \frac{n \pi x}{a} \, dx \]

\[ \int_{0}^{a} \sin \frac{n \pi x}{a} \, dx = \frac{2}{n \pi} \left[ \cos \left( \frac{n \pi x}{a} \right) \right]_{0}^{a} = \frac{2}{n \pi} [\cos(n \pi) - \cos(0)] = \frac{2}{n \pi} [(-1)^n - 1] = \frac{4}{n \pi} \quad \text{for} \ n = 2k \]

\[ \Rightarrow \phi(x, y) = \sum_{n = 2k} \frac{4V}{n \pi} \sin \frac{n \pi x}{a} \left( -\frac{y}{a} \right) \quad \text{for} \ n = \text{even} \]