Reverse statement

If $X(w)$ is analytic in the upper half plane then

$$X(t) = \frac{1}{2\pi i} \int_{\gamma} e^{-iw} X(w) \, dw$$

can close the contour for $t < 0$
gives zero because $X(w)$ is analytic for $\text{Im}(w) > 0$

$$= X(t < 0) = 0$$