2) Multipole Expansion

Finite charge distribution

\( V(r) \) drops off at least as \( \frac{1}{r} \)

We can expand the potential as

\[
\phi(\vec{r}) = \sum_{l=0}^{\infty} P_l(x, y) \frac{1}{r^{l+1}}
\]

the \( l \)th term is called the multipole field of order \( l \).

4) Potential is given by

\[
\phi(\vec{r}) = \int \frac{d\vec{r}' \rho(\vec{r}')}{|\vec{r} - \vec{r}'|}
\]

to get the multipole expansion we must expand \( \frac{1}{|\vec{r} - \vec{r}'|} \) in power of \( \frac{1}{r} \)

\[
\frac{1}{|\vec{r} - \vec{r}'|} = \sum_{l=0}^{\infty} P_l(x, y) \frac{r^l}{r^l l+1}
\]

\[
\sqrt{1 + \frac{r'^2 \cos^2 \theta}{r^2}} = \frac{1}{r} \sqrt{1 + \frac{r^2}{r'^2} \sin^2 \theta}
\]

\( P_l \) are the Legendre polynomials

\[
P_0 = 1
\]

\[
P_1 = \frac{r}{r'}
\]

\[
P_l = \frac{1}{2} (3x^2 - 1)
\]

\( r > r' \) for expansion to be valid