Green's Function

\[ \Box \Psi = -4\pi F \]

Solution

\[ \Psi = \int d^3\vec{r}'d^4t' G(\vec{r},\vec{r}',t,t') F(\vec{r}',t') + X \]

With \( \Box X = 0 \)

\( X \) is determined by boundary conditions

We look for a translational invariant solution

\[ \Box G = -4\pi \delta(0) \delta(t) \]

Fourier transform

\[ G(\vec{k},t) = \int d^3\vec{x} dw e^{i(\vec{k}\cdot\vec{x} - wt)} G(\vec{k},w) \]

\[ \delta(0) \delta(t) = \int d^3\vec{k} dw e^{i(\vec{k}\cdot\vec{r} - wt)} \]

\[ \Leftrightarrow (-\frac{\alpha^2}{c^2} + \omega^2) G(\vec{k},\omega) = -\frac{4\pi}{(2\pi)^3} \]

\[ \Rightarrow G(\vec{k},\omega) = -\frac{1}{4\pi^2} \frac{1}{\frac{\omega^2}{c^2} - k^2} \]

We implement boundary conditions by an ice prescription