Charmonium spectra and dispersion relation with improved Bayesian analysis in lattice QCD

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I. Background

- Strongly coupled QGP
  - Success of ideal hydrodynamic model for QGP at the RHIC
- Dynamical property of QGP medium
  - Real time information is needed
- First-principles calculation (Lattice QCD)
  - Analytic continuation imaginary time to real time is difficult (ill-posed problem)
  - Maximum entropy method

Purpose

1. Improve the error of MEM by extending MEM
2. Analyze the dispersion relation of charmonia at finite temperature.

II. MEM

Analytic continuation

Imaginary time correlation function from Lattice QCD:

\[ D(\tau, \bar{\tau}) = \int d^3 x e^{i \vec{p} \cdot \vec{x}} \left\langle J_1(\tau, \bar{\tau}) J_1^\dagger(0, 0) \right\rangle \]

\[ = \int_0^\infty K(\tau, \omega, \bar{\tau}, \bar{\omega}) \left\langle A(\omega, \bar{\omega}) \right\rangle d\omega \]

MEM

\[ A_{\text{mem}} = \int d\omega \int [A] A(\omega) P(A, \alpha) \]

\[ P(A, \alpha) = \text{[Likelihood function]}(A) \times \text{[Prior probability]}(A, \alpha)/Z \]

The error of MEM

\[ \left\langle (\delta A_{\text{mem}}^2) \right\rangle \]

\[ = \int d\omega \left\langle [A] [A] \right\rangle_{1x1} \delta A(\omega) \delta A(\omega') P(A, \alpha) \]

\[ / \int_1x1 d\omega d\omega' \]

\[ \delta A(\omega) = A(\omega) - A_0(\omega) \]

Usually, the analysis is performed for a single correlation.

III. Improved MEM

Different correlators measured on same configuration have strong correlations:

Take this correlation into Bayesian analysis:

Extend the MEM analysis for the product space of correlation functions:

\[ \text{Conventional MEM} \]

\[ \text{Improved MEM} \]

\[ \text{The correlation is in the covariance matrix } C_{ij} \]

IV. Results

Lattice setup

- SU(3) pure gauge theory
- Wilson gauge and standard Wilson quark action

Spectral function

- Analyze two correlators together
  - The error of spectra is drastically improved.
  - The width of the peak become narrow.
  - The peak correspond to \( \eta_c \) remains at 1.70Tc

Dispersion relation

- The error is estimated as variance of the center of weight.

\[ \text{Variance} = \frac{\text{mean}^2 + \text{mean}}{(\langle \omega A(\omega)/\omega^2 \rangle^2 - \langle A(\omega)/\omega \rangle^2)} \]

- Line: \( \omega = \sqrt{m_0^2 + p^2} \)
- Big improvement at low momentum region

V. Summary & Future plan

- We extend the MEM analysis to the product space of the correlators to take advantage of more data and the strong correlation among Euclidean correlators with different momenta.
- The error of MEM is drastically improved.
- Make \( \alpha \) multi-dimensional.
- More temperature