Analysis of meson screening mass in the entanglement PNJL model

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Introduction

✓ Meson mass is a fundamental quantity to understand the QCD dynamics and the equation of state. At finite temperature (T), the pole mass (temporal-direction mass) is different from the screening mass (spatial-direction mass), because Lorentz symmetry is broken there.
✓ In lattice QCD, the calculation of screening mass is easier than that of pole mass since the imaginary-time direction is limited up to the inverse of T.
✓ In the NJL-type effective models, meanwhile, the calculation of screening mass is much more difficult than that of pole mass, as shown below.
✓ We then propose a way of evaluating the screening mass in the NJL-type effective models in order to compare the model results with lattice QCD ones.

Meson Screening Mass

The screening mass is determined from the mesonic correlator (η) at large distance (r → ∞);
\[ \eta_{\xi}(r) = \frac{1}{4\pi^2 i r} \int_{-\infty}^{\infty} dq \bar{q} q \chi_{\eta}(0, \bar{q}^2) e^{i q r} \sim \frac{1}{r} e^{-M_{\xi,scr} r} \]  

\( \bar{q} \) : External momentum, \( \xi \) : Meson species

\[ \chi_{\xi}(0, \bar{q}^2) : \text{Meson correlation in the momentum space} \]

Model Setting

◆ Model : 2-flavor Entanglement-PNJL (EPNJL) model [1]

\[ L = \bar{q} (i\gamma^\mu D_\mu - m_0) q + G_1(\Phi) \left[ (\bar{q}q)^2 + (\bar{q}\gamma_5 \gamma^\mu q)^2 \right] + U(\Phi, \bar{\Phi}, T) \]

◆ Chiral symmetry breaking and confinement mechanism are included.
◆ The Polyakov-loop potential (U) as a function of Polyakov loop (Φ) is determined from lattice QCD data in pure gauge limit for pressure, entropy, and energy density.
◆ The four-quark coupling strength \( G_1(\Phi) \) depends on Φ. This is called the “entanglement coupling”.

Regularization scheme : Pauli-Villars regularization

This regularization preserves translational invariance in the spatial directions that is important for evaluating \( M_{\xi,scr} \) [2].

Meson correlation \( \chi \) is calculated with the ring approximation.

\[ \chi_{\xi}(0, \bar{q}^2) = \frac{\Pi_{\xi}(0, \bar{q}^2)}{1 - 2\bar{q} \Pi_{\xi}(0, \bar{q}^2)} \]

Difficult and Solution

◆ Difficulty in calculating screening mass

Eq. (1) demands heavy numerical integrations at large distance (r → ∞), since the integrand includes a highly oscillating function \( e^{i q r} \).

In the previous work [2], the calculations were done in the complex \( \bar{q} \)-plane. However, there exist branch cuts in the vicinity of real axis that still requires heavy numerical calculations. (See the left panel of Fig.1)

◆ Our solution

We found the new formalism in which there are no branch cuts in the vicinity of real axis. (See the right panel of Fig.1)

✓ In this flame work, internal momentum is integrated out before taking the summation of Matsubara frequencies.
✓ There is a physical cut starting at \( \bar{q} = i \Delta \), where \( \Delta_T \) is threshold mass:

\[ \Delta_T = \sqrt{M^2 + \left( T - \cos^{-1} \left( \frac{3 - M^2}{2} \right) \right)^2} \]

Screening mass (\( M_{\xi,scr} \)) is defined as a pole in the imaginary axis:

\[ 1 - 2G_1(\Phi, 0, \bar{q}^2) = 0 \]

Results

◆ Pi meson screening mass

✓ LQCD results are below \( M_{th} \) (black line). This indicates that screening mass can be defined as a pole in the complex \( \bar{q} \)-plane.
κ PNJL model (\( s_1 = s_2 = 0 \)) is consistent with lattice QCD only at low temperature (\( T < T_c \)).
κ EPNJL model well reproduces the lattice QCD results for all T. Therefore, the entanglement coupling is essential.

◆ Screening mass and pole masses

✓ Pole masses of pi and sigma mesons are predicted with the EPNJL model.
κ T dependence of pole mass is similar to that of screening mass up to \( T_c \) as a result of chiral symmetry breaking.
κ Above \( T_c \), the pole masses are largely deviated from the corresponding masses. It is then important to use the EPNJL model in order to get the pole masses from the screening masses calculated with lattice QCD.

Summary

✓ We have proposed a practical and reliable method for calculating meson screening masses in NJL-type effective models. The mass is obtained as a pole in complex \( \bar{q} \)-plane.
κ The entanglement coupling is essential to explain T dependence of screening mass calculated with lattice QCD.
κ Below \( T_c \), pole masses are close to screening masses in virtue of chiral symmetry breaking.
κ Above \( T_c \), the pole masses are largely deviated from the corresponding masses. We propose to use the EPNJL model in order to predict the pole masses from the screening masses calculated with lattice QCD.