Anisotropy of the quark-antiquark potential in a magnetic field

C. Bonati¹, M. D’Elia¹, M. Mariti², M. Mesiti³, F. Negro¹, F. Sanfilippo²
¹Dipartimento di Fisica dell’Università di Pisa and INFN, Sezione di Pisa, Pisa, Italy
²School of physics and astronomy, University of Southampton, Southampton, United Kingdom
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Abstract
We study the static quark-antiquark potential in the presence of an external magnetic field, using lattice QCD simulations. The external field breaks the centrality of the potential, which has a different behavior in the directions parallel and perpendicular with respect to the magnetic field. In particular the string tension is larger (smaller) in the perpendicular (parallel) direction. This effect might be relevant for the phenomenology of off-central heavy ion collisions, leading to modifications of the heavy mesons properties, such as the mass spectrum.


QCD in the presence of strong magnetic fields

- Electromagnetic background interacts only with quarks, however loop effects can modify also gluon dynamics.
- Magnetic fields at the strong scale ($eB \approx m_q^2$) can modify the QCD dynamics, leading to relevant non perturbative effects.
- Phenomenological contexts:
  - Class of neutron stars, the magnetars ($eB \approx 10^{10}$ Tesla).
  - Early stage of the Universe ($eB \approx 10^{10}$ Tesla).
  - Off-central heavy ion collisions ($eB \approx 10^{15}$ Tesla).
- Useful to probes QCD properties → Help to clarify many open questions in the theory (chiral and deconfinement transitions, $\theta$-term, etc...).

Off-central heavy ion collisions

- In off-central collisions strong magnetic fields are expected to be created: orthogonal to the reaction plane and almost homogeneous.
- The nuclei contributions to the magnetic field can be evaluated in simple models.\(^7\)
  \[ eB \approx aZ_{\text{EM}} \frac{e}{\alpha} f(b, \tau) \]
  \[ a, \alpha, \zeta \text{ constants, } \zeta \text{ nucleus rapidity, } \]
  \[ f(b, \tau) \text{ function of the impact parameter (b) and proper time (\tau)}. \]
- RHIC→Au-Au collisions with $\sqrt{s} = 200$ GeV per nucleon pair, expected fields up to $eB \approx 0.2$ GeV$^2$.\(^7\)
- LHC→Pb-Pb collisions at center-of-mass energy $\sqrt{s} = 4.5$ TeV per nucleon pair, can reach fields up to $eB \approx 0.3$ GeV$^2$.\(^7\)
- Higher $\sqrt{s}$ gives stronger $B$ fields, but shorter life time → Electrical conductivity could play central role to extend fields duration.\(^7\)

Static $q - \bar{q}$ potential

- $V_{q\bar{q}}$ confining potential: non perturbative property of QCD. For static quarks, well described by the Cornell potential:
  \[ V(r) = \frac{c}{r} + \frac{a}{r^2} \]
  \[ \alpha \text{ Coulomb term, } \sigma \text{ String tension} \]
- From $V(r)$ one qualitatively reproduces spectra of heavy quarkonia.
- On the lattice, measure $V(r)$ from Wilson loop, $W(R, T)$.
  - Create a $q - \bar{q}$ pair at distance $R$.
  - Propagate it for a time interval $T$.
  - Annihilate the pair.
  \[ W(R, T) \approx C \exp \left( - T V(R) \right) \]
- Extract the potential from large time limit:
  \[ V(\hat{r}) = - \lim_{t \to \infty} W(\hat{r}, t) \]
- Static quark limit: $V_{q\bar{q}}$ due only to gauge fields → $B$ fields modify $V_{q\bar{q}}$? $B$ field breaks rotational invariance → Measure $V_{q\bar{q}}$ separating different directions.
- Fix $B = B_z$. Separate transverse (XY) and longitudinal (Z) directions when calculating Wilson loops → Construct the potential along different directions.

Simulation details

- Lattice QCD simulation, with $N_f = 2 + 1$ flavours, physical quark masses and state of art discretization.
- Exploratory study at $T = 0$.

Results

- For each $B$, fit the transverse and longitudinal directions using:
  \[ a V(\hat{r}) = c \hat{d} + \frac{\alpha_d}{\alpha} + \frac{\hat{d} \hat{r}}{\alpha} \]
  \[ \text{where } d = XY, Z. \]
- Extract the Sommer parameter $\alpha_0$ using:
  \[ \alpha_0 = \hat{r} \hat{d} \hat{d} - 1.65 \]
- For each observable $O$, evaluate the ratios for each direction $d$:
  \[ R_{O,d} = \frac{O_\beta}{O(B=0)} \]
- Splitting of these ratios of the order $10 - 20 \%
- Fit $L = 40$ data, according to:
  \[ R_{O,d} = 1 + A^{d} (|eB|) C^{O,d} \]

Conclusions

- External $B$ fields induce relevant anisotropy in the static $V_{q\bar{q}}$ starting from $eB \approx 0.2$ GeV$^2$.
- With respect to $B$ orientation, $\sigma$ increase (decrease) in the transverse (longitudinal) directions, $\tau_0$ and $\alpha$ show an opposite behavior.
- Investigate $V_{q\bar{q}}$ complete angular dependence with respect to $B$.
- Finite temperature case → relevant for off-central heavy ion collisions.
- Study of the flux tube profile along different directions.

References