$U(1)_A$ symmetry at finite temperature with DWF

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Low temperature – symmetries

\( SU(2)_L \times SU(2)_R \times U(1)_V \times U(1)_A \)

- **Dirac operator eigenmodes**
- **Near zero modes density**
  \[ \Sigma = \pi \rho(0) \]
- **Zero modes**
  \[ \int \partial_\mu J_{\mu 5} \propto Q \]
High temperature – symmetries

\[ SU(2)_L \times SU(2)_R \times U(1)_V \times U(1)_A \]

Current knowledge

**Restoration** of chiral symmetry at \( T_c \)

**Restoration** at \( T \to \infty \)

No condensate

Axial symmetry

\( T > T_c \)
High temperature – symmetries

\[ SU(2)_L \times SU(2)_R \times U(1)_V \times U(1)_A \]

\[ T \geq T_C ? \]

No condensate
Axial symmetry?

\[ T > T_C \]
Methods & Results

Work in progress

U(1)$_A$

symmetry

Finite temperature

Final thoughts

Literature
Recent literature - I

G. Cossu et al. (2013) for JLQCD
Disconnected meson diagrams vanish at temperatures above $T_c$

Related: Gap in the Dirac spectrum

Aoki, Fukaya, Taniguchi (2012)
Analytic calculation (Overlap)
Dirac spectrum $\rho(\lambda) \sim c\lambda^3$
Implies $U(1)_A$ anomaly invisible

Meson correlators

$\beta = 2.25 (T \sim 192) \text{ am}= 0.01$

$\pi = \delta = \rho = \sigma$

Restored
Recent literature - II

Bazavov et al. (2012-13)
Domain wall, several volumes
Dirac spectrum, susceptibilities
**NOT restored**

Ohno et al., Sharma et al. (2012-13)
Overlap on HISQ configurations
Dirac spectrum
**NOT restored**

Brandt et al. (2013)
Wilson improved fermions
Screening masses
**NOT restored**

Our previous study
**Exact** chiral symmetry (Overlap)
**topology fixed**
Only $16^3 \times 8$ volume
Mass dependence
No continuum limit
U(1)\textsubscript{A} symmetry
Finite temperature

Methods & Results

Work in progress

Final thoughts
Generalized Domain Wall

\[ D^4(m) = \frac{1 + m}{2} + \frac{1 - m}{2} \gamma_5 \text{sgn}(H) \]

Play with the sign function

Möbius Kernel

\[ H_M = \gamma_5 \frac{b \Omega_W}{2 + c \Omega_W} \]

Function approximation
Transfer matrix in 5D
- Hyperbolic tangent
- Rational approximation

Reduced residual mass
\[ b=2 \ c=1 \text{ Scaled Shamir, } \mathbf{m}_{\text{res}} \sim 10^{-4} \]
Status of simulations

Symanzik + smeared DWF
Multipurpose code, HMC & measurements
Available on request, soon online
Optimized for BlueGene/Q
Webpage: http://suchix.kek.jp/guido_cossu/

Collected data
- 2 volumes
- 3 masses
- 5 temperatures
- Topology changes
- $N_t=8$, $N_t=12$

Full analysis in progress
Phase transition

Today:
\[ T = 184, 200 \text{ MeV} \] (red arrows)

Phase transition at \(~180 \text{ MeV}\)

2 volumes
Mass dependence

\[ N_t = 12 \] analysis not ready
Using local source is **dangerous**

**Stochastic** measurement is in nice accordance with the **spectral sum**

\[
\Delta = \int \frac{2m^2 \rho(\lambda, m)}{(\lambda^2 + m^2)^2}
\]

\[
\Delta = \chi_\pi - \chi_\delta
\]

\[
\chi_X = \int \langle X(0)X(x) \rangle
\]
Source of the signal

Discrete spectral sum

\[
\Delta = \frac{2N_0}{Vm^2} + \sum_{\lambda \neq 0} \frac{2m^2}{V(\lambda^2 + m^2)^2}
\]

Zero modes

Peaks dominate the signal

76%

Fluctuations of 3 orders of magnitude

One mode signal
\(16^3 \times 8 \beta = 4.10\) am=0.01 Ls=12

Stochastic - Q from Wilson Flow

**Mild correlation**

**Tension** with spectral sum expectations

Two sources
- GW violations
- \( FF \tilde{F} \) estimate

Q=0 near zero modes
Temperature dependence

Broad picture arising at this stage:

- **Just above** the phase transition zero modes dominate
- **Then** they are **strongly suppressed** and the signal goes down
Let’s increase volume – $m=0.01$

Zero mode contribution suppressed $\sim 1/V$
As expected from spectral sum

Bulk contribution increases

Decrease the mass?
Let’s increase volume – $m=0.005$

Zero mode contribution suppressed $\sim 1/V$
As expected from spectral sum

Bulk contribution increases
Volume & mass dependence

$$\chi_t = \lim_{V \to \infty} \frac{\langle Q^2 \rangle}{V} = \text{const.} \rightarrow \frac{N_0}{V} \rightarrow 0$$

Zero modes contribution must vanish

Conclusion: **signal from the bulk part, near zero modes**

Let’s cut all configurations with Q>0

Signal **constant with the mass**
Is everything all right? – I

From the Ginsparg-Wilson relation we can measure the amount of violation for each mode, $g_{kk}$

$$\frac{1}{1 - m^2} \left[ (\gamma_5 - H_m)(H_m - m\gamma_5) + (H_m - m\gamma_5)(\gamma_5 - H_m) \right] = 0$$

$$g_{kk}^m = \psi_m^k \gamma_5 \psi_m^k - \frac{(\lambda_m^k)^2 + m}{\lambda_m^k (1 + m)}$$

$$m_{\text{res}} = \frac{\sum_k \frac{(1+m)}{(1-m)^2 \lambda_m^k} g_{kk}^m}{\sum_k \frac{1}{(\lambda_m^k)^2}}$$
Is everything all right? – II

Lowest modes show violations of GW by 1 order of magnitude bigger than the average

\[ m_{\text{res},k} = \frac{(1+m)\lambda_m^k g_{kk}^m}{\sum_k \left(\frac{1}{\lambda_m^k}\right)^2} \]
Reweight it! (DWF to Overlap)

Before

200 MeV

$16^3 \times 8 \beta = 4.10$ am=0.01 Ls=12
Stochastic - Q from Wilson Flow

After

$16^3 \times 8 \beta = 4.10$ am=0.01 Ls=12
Spectral Sum reweighted to OV

76% 85%
Reweighting alters the final answer!

$16^3 \times 8 \beta = 4.10 \ am = 0.01 \ Ls = 12$

SpectralSum OV - $Q$ from $f_i$

Not reweighted

Just Overlap eigenvalues

Reweighted
Temperature and mass dependence

$T \sim 184$ MeV Just above $T_c$

Quark mass 10 times smaller

$16^3 \times 8 \beta = 4.07$ am=0.01 Ls=12
Spectral Sum reweighted to OV

$16^3 \times 8 \beta = 4.10$ am=0.001 Ls=24
Spectral Sum reweighted to OV

Agrees with Overlap result

$\sim 3$ orders of magnitude smaller signal

Zero in chiral limit?
Methods & Results

Work in progress
Instanton gas – hints?

Results not yet conclusive (analysis running right now)

If the large volume signal is not coming from lattice artifacts
Near zero modes are responsible for breaking $U(1)$

What are they? Poisson distributed?
Fun with 3D – put your glasses on

Chirality

Norm

Eigenvalue

IPR ~34

instanton

anti-instanton
Fun with 3D – put your glasses on

Chirality

Norm

Eigenvalue

IPR ~34
Fun with 3D – put your glasses on

**Chirality**

**Norm**

IPR ~5

Eigenvalue
Summary – one more slide...

- DWF volume & mass dependence suggests near zero modes are the source of U(1) breaking.

- Lattice artifacts can spoil the signal.

- DWF lowest modes look like an instanton weakly interacting gas.

- Exact chiral symmetry results differ from DWF.
Are we finished?

The talk is over the work is not!

Collected data yet to analyze
- Reweighting
- Continuum limit
- Chiral limit

Lattice artifacts?
Gas of instanton pairs, dyons?
Correlation with Polyakov loop?
U(1) restoration above critical temperature is still an open question.
Thanks!

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Backup slides
Lowest mode vs Polyakov Loop
GW violations

$16^3 \times 8 \beta = 4.10 \text{ am}=0.01 \text{ L}=12$
Many configurations violate GW
DW – OV eigenvalue mismatch

$32^3 \times 8 \quad 4.10 \quad 0.005$
Susceptibility scales with volume
Let’s increase volume

Zero mode contribution $\sim 1/V$ - Bulk contribution increases