Anomalous hydrodynamic simulations of heavy-ion collisions
[arxiv:1309.2823]

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Anomaly-induced transport effects

\[ \vec{j} = \kappa_B \vec{B} \quad \vec{j}_5 = \xi_B \vec{B} \]

\[ \vec{j} = \kappa_\omega \vec{\omega} \quad \vec{j}_5 = \xi_\omega \vec{\omega} \]
Realized in heavy-ion collisions?

hydrodynamic expansion

observed as hadrons
Realized in heavy-ion collisions?

Electromagnetic fields & anomalous hydrodynamics

Observed as hadrons
Anomalous hydrodynamic model

**Initial condition**
- Smooth Optical Glauber
- Lumpy (event-by-event) MC Glauber

**Hydrodynamic evolution**
- Anomalous perfect fluid
- Expanding ($\tau$-$\eta$) coordinate
- EM fields treated as background

**Hadronization**
- Cooper-Frye formula applied at freezeout $T$
- MC sampling for event-by-event sim.
Anomalous hydrodynamic model

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Hadronization
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Chiral magnetic wave \[ \mathbf{j} = \frac{e^2 \mu_5}{2\pi^2} \mathbf{B} \]

\[ \mathbf{j}_5 = \frac{e^2 \mu}{2\pi^2} \mathbf{B} \]

\( \mathbf{B} \)

\( \mu > 0 \) (Stopped charge)

\[ \mu_5 > 0 \]
\[ \mu_5 < 0 \]

[Burnier, Kharzeev, Liao, & Yee (2011)]
Axial charge generation from $E \cdot B$

\[
\partial_\mu j_5^\mu = C E_\mu B^\mu
\]

\[\vec{E} \cdot \vec{B} > 0\]

\[\vec{E} \cdot \vec{B} < 0\]

\[\mu = 0\]

\[\mu_5 > 0\]

\[\mu_5 < 0\]
Harmonics $\mathcal{V}_n$

- Azimuthal angle distribution of observed particles

$$\frac{dN}{d\phi} = \bar{N} \left[ 1 + \sum_{n=1}^{\infty} 2\mathcal{V}_n \cos n(\phi - \Psi_n) \right]$$

- Represents the shape of the flow

$\mathcal{V}_2$ “elliptic” $\mathcal{V}_3$ “triangular”
Observables in heavy ion collisions

\[ v_2^- > v_2^+ \]

\[ v_2^\pm = v_2 \pm \frac{rA}{2} \]

\[ A = \frac{N_+ - N_-}{N_+ + N_-} \]

[Burnier, Kharzeev, Liao, & Yee (2011)]
Observables in heavy ion collisions

\[ v_2^\pm = v_2 \mp \frac{r A}{2} \]

\[ A = \frac{N_+ - N_-}{N_+ + N_-} \]

[STAR, ALICE]


[Belmont, QM14]
Equations of anomalous hydrodynamics
Anomalous hydrodynamics equations

- Two U(1) currents (vector & axial vector)
- Perfect fluid
- Background electromagnetic fields

\[
\partial_\mu T^{\mu\nu} = F^{\nu\rho} j_\rho
\]

\[
\partial_\mu j^\mu = 0 \quad \partial_\mu j_5^\mu = CE_\mu B^\mu
\]

\[
C = \frac{N_c}{2\pi^2} \quad E^\mu = F^{\mu\nu} u_\nu \quad B^\mu = \tilde{F}^{\mu\nu} u_\nu
\]
Anomalous hydrodynamics equations

- Constitutive equations

\[
\begin{align*}
\dot{j}^\mu &= n u^\mu + \kappa_B B^\mu \\
\dot{j}_5^\mu &= n_5 u^\mu + \xi_B B^\mu \\
\omega^\mu &= \frac{1}{2} \varepsilon^{\mu\nu\rho\sigma} u_\nu \partial_\rho u_\sigma
\end{align*}
\]

- Transport coefficients

\[
\begin{align*}
e \kappa_B &= C \mu_5 \left(1 - \frac{\mu_5 n_5}{e + p}\right) \\
e \xi_B &= C \mu \left(1 - \frac{\mu n}{e + p}\right) \\
e^2 \kappa_\omega &= 2 C \mu \mu_5 \left(1 - \frac{\mu n}{e + p}\right) \\
e^2 \xi_\omega &= C \mu^2 \left(1 - \frac{2 \mu_5 n_5}{e + p}\right)
\end{align*}
\]
Time evolution of anomalous fluids
Initial condition & EM fields

Optical Glauber initial condition
- impact parameter: 7.2 fm

Initial chemical potential is given so that $n/s = \text{const.}$

Electromagnetic fields

$$B_y(\tau, x_\perp) = B_0 \exp \left[ -\frac{x^2}{\sigma^2_x} - \frac{y^2}{\sigma^2_y} \right] \exp \left[ -\frac{\tau}{\tau_B} \right]$$

$$E_y(\tau, x_\perp) = \frac{y}{y_0} \times E_0 \exp \left[ -\frac{x^2}{\sigma^2_x} - \frac{y^2}{\sigma^2_y} \right] \exp \left[ -\frac{\tau}{\tau_E} \right]$$

$eB_0 = 0.08 \text{ GeV}^2$  \quad $eE_0 = 0.02 \text{ GeV}^2$

$\tau_B = 3 \text{ fm}$  \quad $\tau_E = 2 \text{ fm}$
- Optical Glauber
- Anomalous
- Initially $\mu = 0$
- Optical Glauber
- $C = 0$
- Initially $\mu = 0$
HIC-like initial condition

- Monte-Carlo Glauber initial condition
  - impact parameter: 7.2 fm
- MC-Glauber
- Anomalous
- Initially $\frac{\mu}{T} = 0$
Particle distributions

- Cooper-Frye formula

\[
E \frac{dN}{d^3p} = \frac{d}{(2\pi)^3} \int \sum \frac{p \cdot d\sigma}{\exp[(p \cdot u - \mu)/T]} +_{BF} 1
\]

- Freezeout temperature: 160 MeV

- Obtain the distributions of \( N_\pm \rightarrow \Delta v_2 \equiv v_2^- - v_2^+ \)
Charge-dependent $\nu_2$

- Linear in charge asymmetry, slopes are almost the same
- Different intercepts for $C=0$ and $C\neq 0$
Why slope is finite even for $C=0$?

- More positive particles from the central part
Why slope is finite even for $C=0$?

- More positive particles from the central part
- In total, $v_2^- > v_2^+$
- Background to signal of anomalous transport effects
Summary

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Backup slides
Hydrodynamics equations

- Equation of state
  - non-interacting massless 1-flavor fermion & gluons

\[ p = \frac{1}{3} e = \frac{g \pi^2}{30} T^4 + \frac{N_c}{6} (\mu^2 + \mu_5^2) T^2 + \frac{N_c}{12\pi^2} (\mu^4 + 6\mu^2 \mu_5^2 + \mu_5^2) \]

\[ g = \frac{7}{8} g_q + g_g \quad N_c = 3 \]
- MC-Glauber
- Anomalous
- initially $\frac{\mu}{T} = 0.01$
Electromagnetic fields in HIC

- Magnetic fields in off-central collisions
  - $e|\vec{B}| \sim m_{\pi}^2 \sim 10^{14}$ T
  - Neutron star: $10^8$ T
  - Magnetar: $10^{11}$ T

- Electric fields also exist
  - $e|\vec{E}| \sim m_{\pi}^2$
  - [A. Bzdak and V. Skokov, Phys. Lett. B 710, 171 (2012)]
  - …