Fluctuations of conserved charges in QCD

BNL-Bi-CCNU Collaboration:
Critical Point and Onset of Deconfinement

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Scientific Topics:
- Critical Point
- Phase transitions in hot and dense matter
- Deconfinement and chiral symmetry restoration
- Hadronization and chemical freeze-out
- Compact Stars
- Future facilities, detectors and methods

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Expansion of the pressure:

\[
\frac{p}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{BQS}^{ijk,0} \left( \frac{\mu_B}{T} \right)^i \left( \frac{\mu_Q}{T} \right)^j \left( \frac{\mu_S}{T} \right)^k
\]

\[X = B, Q, S: \text{conserved charges}\]

Lattice

\[
\chi^X_n = \left. \frac{\partial^n [p/T^4]}{\partial (\mu_X/T)^n} \right|_{\mu_X=0}
\]

generalized susceptibilities

\[\Rightarrow \text{only at } \mu_X = 0!\]

Experiment

\[
VT^3 \chi^X_2 = \langle (\delta N_X)^2 \rangle
\]
\[
VT^3 \chi^X_4 = \langle (\delta N_X)^4 \rangle - 3 \langle (\delta N_X)^2 \rangle^2
\]
\[
VT^3 \chi^X_6 = \langle (\delta N_X)^4 \rangle - 15 \langle (\delta N_X)^4 \rangle \langle (\delta N_X)^2 \rangle + 30 \langle (\delta N_X)^2 \rangle^3
\]

cumulants of net-charge fluctuations

\[\delta N_X \equiv N_X - \langle N_X \rangle\]

\[\Rightarrow \text{only at freeze-out } (\mu_f(\sqrt{s}), T_f(\sqrt{s}))!\]
Motivations

Explore the QCD phase diagram

• Analyze higher order cumulants and test universal scaling behavior
  → make prediction on the radius of convergence and possible experimental observables

Analyze freeze-out conditions

• Match various cumulant ratios of measured fluctuations to QCD
  → determine freeze-out parameter


Identify the relevant degrees of freedom

• Compare (lattice) QCD fluctuations to various hadronic/quasiparticle models:
  → deconfinement vs. chiral transition (melting of open strange/charm hadrons)
  → evidence for experimentally not yet observed hadrons

BNL-Bielefeld, PRL 111 (2013) 082301;
Lattice parameters:

- (2+1)-flavor of highly improved staggered fermions (HISQ-action)
- a set of different lattice spacings \( N_\tau = 6, 8, 12 \)
- two different pion masses: \( m_\pi = 140, 160 \text{ MeV} \)
- high statistics: \((10 - 16) \times 10^3\) configurations

\( \Rightarrow \) statistical and systematical errors are under control
\( \Rightarrow \) In general: find good agreement with HRG model for \( T < 155 \text{ MeV} \)
Observables: traces of combinations of $M^{-1}$ and $M^{(n)} = \partial^n M / \partial \mu^n$

$$\frac{\partial \ln Z}{\partial \mu} = \frac{1}{Z} \int D U \ Tr \ [M^{-1} M'] \ e^{Tr \ln M} e^{-\beta S_G}$$

$$= \langle Tr \ [M^{-1} M'] \rangle$$

$$\frac{\partial^2 \ln Z}{\partial \mu^2} = \langle Tr \ [M^{-1} M''] \rangle - \langle Tr \ [M^{-1} M' M^{-1}] \rangle + \langle Tr \ [M^{-1} M']^2 \rangle$$

Method: stochastic estimators with $N = 1500$ random vectors

$$Tr [Q] \approx \frac{1}{N} \sum_{i=1}^{N} \eta_i^\dagger Q \eta_i$$

with

$$\lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} \eta_i^\dagger \eta_i = \delta_{x,y}$$
Freeze-out conditions in HIC
Apply: initial conditions in HIC

- strangeness neutrality: $\langle N_S \rangle = 0$
- isospin asymmetry: $\langle N_Q \rangle = r \langle N_B \rangle$

Expand in powers of $\mu_B, \mu_Q, \mu_S$
solve for $\mu_Q, \mu_S$

$$\mu_Q(T, \mu_B) = q_1(T)\hat{\mu}_B + q_3(T)\hat{\mu}_B^3$$
$$\mu_S(T, \mu_B) = s_1(T)\hat{\mu}_B + s_3(T)\hat{\mu}_B^3$$

$\hat{\mu}_B = \mu_B/T$

two independent parameter remain: $T^f, \mu_B^f$

need two ratios of cumulants from experiment to fix

$$\mu_Q^f \equiv \mu_Q(T^f, \mu_B^f)$$
$$\mu_S^f \equiv \mu_S(T^f, \mu_B^f)$$
• Need two ratios of cumulants to fix the remaining two freeze-out parameters $T^f$, $\mu_B^f$

⇒ consider ratios of cumulants of **electric charge fluctuations**

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**Experiment:**

$$\frac{M_Q(\sqrt{s})}{\sigma_Q^2(\sqrt{s})} = \frac{\chi^Q_1(T, \mu_B)}{\chi^Q_2(T, \mu_B)} = R^Q_{12}(T, \mu_B)$$

LO linear in $\mu_B$, fixes $\mu_B^f$

(baryometer)

$$\frac{S_Q(\sqrt{s})\sigma_Q^3(\sqrt{s})}{M_Q(\sqrt{s})} = \frac{\chi^Q_3(T, \mu_B)}{\chi^Q_1(T, \mu_B)} = R^Q_{31}(T, \mu_B)$$

LO independent of $\mu_B$, fixes $T^f$

(thermometer)

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**Lattice:**

$M$: mean

$\sigma$: variance

$S$: skewness

$\chi_n$: generalized susceptibilities

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difficult to determine $\sqrt{s}$-dependence of $T^f$, need more precise experimental data from BES (BES-II)

⇒ need to check self-consistence using other ratios as input
Relevant degrees of freedom
Some notable differences in the strangeness sector

\[ p_{m_i}^{H}{(T, \mu_S)} + \sum_{i\in \text{baryons}} p_{m_i}^{B}{(T, \mu_B, \mu_S)} \]

hadron masses from PDG up to 2.5 GeV, and $p_{m_i}^{M/B}$ pressure of free bosonic/fermionic quantum gas
Also apparent in B,S off-diagonal cumulants

\[ \Rightarrow \text{overshooting of HRG in the crossover region and below} \]
The QM-HRG

Are differences due to missing states in the PDG?
- Obvious in the charm sector
- How large could be the effect of missing states in the strange sector?

⇒ construct QM-HRG, including additional states predicted by Quark-Model

- Use mesonic states from: S. Capstick and N. Isgur, PRD 34, 2809 (1986).
- Use baryonic states from: D. Ebert et al., PRD 79, 114029 (2009)

- Similar to the spectrum of strange baryons on the lattice
Partial pressure of strange mesons and baryons:

- Boltzmann approximation is used here and in the following

\[
P_{M/B}^{S,X}(T, \bar{\mu}) = \frac{T}{2\pi^2} \sum_{i \in X} g_i \left( \frac{m_i}{T} \right)^2 K_2(m_i/T) \cosh(B_i \bar{\mu}_B + Q_i \bar{\mu}_Q + S_i \bar{\mu}_S)
\]

with \( X = \text{PDG}, \text{QM} \) and \( \bar{\mu}_q = \mu_q/T, q = B, Q, S \)

⇒ open strange baryon sector experimentally much less known, additional baryons contribute up to 30% at \( T=170 \) MeV

⇒ open strange meson sector experimentally well known
Evidence for more strange hadrons

- **BS-correlation** $\chi_{11}^{BS}$ at low $T$: weighted sum of partial pressure of strange baryons

- Different **linear combinations** of $\chi_2^S, \chi_4^S, \chi_{11}^S, \chi_{31}^S, \chi_{22}^S, \chi_{13}^S$ are used to project onto partial pressure of strange baryons ($B_i^S$) and mesons ($M_i^S$) in the hadronic phase, e.g.

  \[
  B_1^S = -\frac{1}{6} (11\chi_{11}^{BS} + 6\chi_{22}^{BS} + \chi_{13}^{BS}) \\
  B_2^S = \frac{1}{12} (\chi_4^S - \chi_2^S) - \frac{1}{3} (4\chi_{11}^{BS} - \chi_{13}^{BS})
  \]

⇒ **QM-PDG** provides more accurate description of lattice data

⇒ Re-confirmation of our previous findings [PRL 111,082301]: onset of melting of open strange hadrons consistent with chiral crossover
QM-HRG for the charm sector

charged charm baryons to charmed mesons

strange charm baryons to charmed mesons

⇒ evidence for more charmed hadrons

⇒ melting of open charm hadrons start at chiral crossover

Probing hadron spectrum using QCD thermodynamics

Padmanath et. al.: arXiv:1311.4806 [hep-lat]

P̂ all PDG states

\[ \text{tot} = \text{all hadrons} \]

LQCD̂ \( + \) ???

Ebert et. al.: Eur. Phys. J. C66, 197 (2010);

Christian Schmidt                     XQCD 2014, SUNY Stony Brook, June 20, 2014

Figure 3: Ratios of baryon-electric charge
to charmed baryons, for different charm sectors in the numerator, with different contributors. The ratios shown in Fig. 4 thus provide first-principles evidence for more charmed hadrons. The observables shown in Fig. 4 thus provide first-principles evidence for more charmed hadrons.
coming back to ...

Freeze-out conditions in HIC

Now determined from relative yields of open strange hadrons.
Implications for freeze-out conditions

- in order to make contact to experiment: apply strangeness neutrality constraint $n_S = 0$ (and iso-spin $n_Q/n_B = 0.4$)

$\Rightarrow T, \mu_B, \mu_S, \mu_Q$ not independent

(Lattice) QCD: unique expansion

$$(\mu_S/\mu_B)(T) = s_1(T) + s_3(T)\hat{\mu}_B^2 + \mathcal{O}(\hat{\mu}_B^4)$$

$$\Rightarrow \left(\frac{\mu_S}{\mu_B}\right)_{LO} \equiv s_1(T) = -\frac{\chi^{BS}_{11}}{\chi^S_2} - \frac{\chi^{QS}_{11}}{\chi^S_2} \frac{\mu_Q}{\mu_B}$$

- small corrections from $\mu_Q > 0$
- neglect NLO contribution, known to be small for $\mu_B \lesssim 200 \text{ MeV}$ [PRL 111, 082301]

HRG: $\mu_S(T, \mu_B)$ depends on relative abundance of open strange baryons and mesons

additional strange baryons

$\Rightarrow$ larger $\mu_S$

(at fixed $T, \mu_B$)
Implications for freeze-out conditions

Experiment:
relative yields of strange anti-baryons to baryons at freeze-out are controlled by freeze-out parameter \((T^f, \mu_B^f, \mu_S^f)\)

\[
\frac{n_{\bar{B}}}{n_B} = \exp \left\{ - \frac{2\mu_B^f}{T^f} + \frac{2\mu_S^f}{T^f}|S| \right\} \\
= \exp \left\{ - \frac{2\mu_B^f}{T^f} \left( 1 - \frac{\mu_S^f}{\mu_B^f}|S| \right) \right\}
\]

\(\Rightarrow\) \(\mu_B^f/T^f\) and \(\mu_S^f/\mu_B^f\) can be obtained by fitting experimentally measured values of \(\bar{\Lambda}/\Lambda\), \(\bar{\Xi}/\Xi\) and \(\bar{\Omega}/\Omega\)

\(\Rightarrow\) fit function independent of details of open strange hadron spectrum
Implications for freeze-out conditions

Comparison:

vary $T_f$ in order to match $\mu^f_B / T^f$ and $\mu^f_S / \mu^f_B$

$\Rightarrow$ QM-HRG in agreement with (lattice) QCD

$\Rightarrow$ QM-HRG yields 5-8 MeV smaller freeze-out temperature than PDG-HRG
Summary and Conclusions

• ratios of fluctuations of conserved charges can be used to determine the freeze-out parameter in HIC

• partial pressure of strange baryons from QM-HRG and PDG-HRG differ by about 30% at T=170 MeV (even more for charmed baryons)

  ⇒ additional strange/charm baryons are thermodynamically relevant in the crossover region

• in the crossover region the QM-HRG provides a more accurate description of (lattice) QCD w.r.t. the conventionally used PDG-HRG

  ⇒ evidence for additional, experimentally not yet observed, strange/charm baryons

• presence of additional strange hadrons get imprinted in the yields of ground state strange hadrons

  ⇒ significant reductions of freeze-out temperature of (5-8) MeV if determined from relative yields of strange hadron ($\mu_S/\mu_B$)
Make contact with experiment

**Figure 7:** Freeze-out temperatures $T_f$ and baryon chemical potentials $\mu_f$ obtained through direct comparisons between LQCD calculations and the preliminary STAR and PHENIX data for cumulants of net charge and net proton fluctuations. The shaded region indicates the LQCD results \[1, 4, 7\] for the chiral/deconfinement temperature $T_c$ as a function of the baryon chemical potential.

Despite the caveat that cumulants of net proton fluctuation may be quantitatively different from the cumulants of net baryon number fluctuation \[11, 12\], in Fig. 6(b) we present a comparison between the LQCD results for $R_B^{12}$ and the preliminary STAR data \[13\] for the ratio $M_p/s_p^2$ of the cumulants of net proton fluctuation. Unfortunately, as can be seen from Fig. 5(b) and Fig. 6, the freeze-out baryon chemical potential obtained from all these experimental measurements are not consistent with each other at present. Furthermore, they are also not consistent with the freeze-out baryon chemical potential obtained from the traditional statistical model fits \[14\]. To illustrate this more clearly in Fig. 7 we show the freeze-out parameters $T_f$ and $\mu_f$ extracted by comparing LQCD calculations with the preliminary STAR and PHENIX results for the cumulants of net charge fluctuations as well as with the preliminary STAR data for the cumulants of net proton fluctuations. While these results differ from each other and from that obtained using the statistical model fits \[15\], it is tantalizing to see that all these results lie within the chiral/deconfinement crossover region,

$$T_c(\mu_B) = \begin{cases} 154 \pm 9 \text{ MeV} & (8.0066 \pm 7) / 154 \end{cases}$$

obtained from LQCD calculations \[1, 4, 7\]. This makes us hopeful that the HIC collision experiments may signal presence of criticality in the QCD phase diagram in the $T - \mu_B$ plane.

While such direct comparisons between the LQCD calculations and HIC experiments may open up many new opportunities, at present, one has to be somewhat cautious. The LQCD calculations of generalized susceptibilities are performed using a grand-canonical ensemble approach in the thermodynamic limit. It is a-priori not evident that this is also applicable to conditions met in a heavy ion collision. Thus while comparing our results with experimental ones we must make sure that effects of conservation laws due to finite system sizes, acceptance cuts \[11, 16\] etc. do not invalidate the grand canonical ensemble approach. These questions are currently being addressed in experimental analysis \[9, 10, 13\] and hopefully will be resolved soon.

Mukherjee, Wagner, CPOD’13